In the Oceans of Water Air & Ether

THE

WAVE OF TRANSLATION

IN THE

OCEANS OF WATER, AIR, AND ETHER.

BY

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PREFACE.

Of the three papers now submitted to that portion of the reading public which does not shrink from some degrèe of mental effort, two see the light for the first time. They were prepared for the Royal Society of London in 1881, the author hoping to have been able to have read them there himself, with a view to subsequent publication in the Transactions of the Society; but a continued illness obliged him to renounce this expectation, and it remained to his family to carry out his wish as they best could, in order that his later researches and speculations in physical science should not be lost, though he himself could not superintend their issue. To this circumstance must be attributed whatever defects or shortcomings may be found in the papers as now published. These remarks apply with special force to the paper on Ether.

After these two papers were sent in to the Royal Society, it was suggested to him, that without some previous knowledge of the researches which he had formerly made, between the years 1833 and 1840, in Hydrodynamics, and notably in the nature, character,

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PART I.

THE WAVE OF TRANSLATION.

UNTIL the date of the observations which I made at the request of the British Association, there had been confounded together in an indefinite notion of waves and wave motion, phenomena essentially different—different in their genesis, laws of propagation, and other characteristics. I have endeavoured, by a rigid course of examination, to distinguish these different classes of phenomena from each other. I have determined certain tests, by which these confused phenomena have been made to divide themselves into certain classes, distinguished by certain great characteristics. Contradictions and anomalies have in this process gradually disappeared; and I now find that all the waves which I have observed may be distinguished into four great orders, and that the waves of each order differ essentially from each other in the circumstances of their origin, are transmitted by different forces, exist in different conditions, and are governed by different laws. It is now therefore easy to understand how much has been hitherto added to the difficulty of this difficult subject, by confounding together phenomena so different. The characteristics, phenomena, and laws of these great orders, I have attempted in the present report to determine and define.

The knowledge I have thus endeavoured to obtain and herein to set forth concerning these beautiful and interesting wave phenomena, is designed to form a contribution to the advancement of hydrodynamics, a branch of physical science hitherto much in arrear. But besides this their immediate design, these investigations of wave motion are fertile in important applications, not only to illustrate and extend other departments of science, but to subserve the purposes and uses of the practical arts. I have ascertained that what I have called the great wave of translation, my wave of the first order, furnishes a type of that great oceanic wave which twice a day brings to our shores the waters of the tide.

This application of the phenomena of waves to explain the tides is not their only application to the advancement of other branches of science. The phenomena of *resistance of fluids* I have found to be intimately connected with those waves. The resistance which the water in a channel opposes to the passage of a floating body along that channel depends materially on the nature of the great wave of the first order.

If to these two branches of science we add the useful arts, in which an accurate acquaintance with wave phenomena may be of practical value to the purposes of human life, we shall find that the improvement of *tidal rivers*, the construction of *public works* exposed to the action of waves and of tides, and the *formation of ships*, are among the most direct and necessary applications of this knowledge, which is indeed essential to the just understanding of the best methods of opposing the violence of waves, and converting their motion to our own uses.

The Nature of Waves and their Variety.

When the surface of water is agitated by a storm, it is difficult to recognise in its tumultuous tossings any semblance of order, law, or definite form, which the mind can embrace so as adequately to conceive and understand. Yet in all the madness of the wildest sea the careful observer may find some traces of method; amid the chaos of water he will observe some moving forms which he can group or individualise; he may distinguish some which are round and long, others that are high and sharp; he may observe those that are high gradually becoming acuminate and breaking with a foaming crest, and may notice that the motion of those which are small is short and quick, while the rising and falling of large elevations is long and slow. Some of the crests will advance with a great, others with a less velocity; and in all he will recognise a general form familiar to his mind as the form of the sea in agitation, and which at once distinguishes it from all other phenomena.

Just as the waters of a reservoir or lake when in perfect repose are characterised by a smooth and horizontal surface, so also does a condition of disturbance and agitation give to the surface of the fluid this form characteristic of that condition, and which we may term the wave form. When any limited portion of the wave surface presents a defined figure or boundary, which appears to distinguish that portion of fluid visibly from the surrounding mass, our mind gives it individuality—we call it *a wave.*

It is not easy to give a perfect definition of a wave, nor clearly to explain its nature so as to convey an accurate or sufficiently general conception of it. Persons who are placed for the first time on a stormy sea, have expressed to me their surprise to find that their ship, at one moment in the trough between two waves, with every appearance of instant destruction from the huge heap of waters rolling over it, was in the next moment riding in safety on the top of the billow. They discover with wonder that the large waves which they see rushing along with a velocity of many miles an hour, do not carry the floating body along with them, but seem to pass under the bottom of the ship without injuring it, and indeed with scarcely a perceptible effect in carrying the vessel out of its course. In like manner the observer near the shore perceives that pieces of wood, or any floating bodies immersed in the water near its surface, and the water in their vicinity, are not carried towards the shore with the rapidity of the wave, but are left nearly in the same place after the wave has passed them, as before. Nay, if the tide be ebbing, the waves may even be observed coming with considerable velocity towards the shore, while the body of water is actually receding, and any object floating in it is carried in the opposite direction to the waves, out to sea. Thus it is that we are impressed with the idea, that *the motion of a wave may be different from the motion of the water* in which it moves; that the water may move in one direction and the wave in another; that water may transmit a wave while itself may remain in the same place.

If then we have learned that a water wave *is not* what it seems, a heap of water moving along the surface of the sea with a velocity visible to the eye, it is natural to inquire what a wave really is; *what is wave motion as distinct from water motion?*

For the purpose of this inquiry let us take a simple example. I have a long narrow trough or channel of water, filled to the depth of my finger length. I place my hand in the water, and for a second of time push forward along the channel the water which my hand touches, and instantly cease from further motion. The immediate result is easily conceived; I have simply pushed forward the particles of water which I touched, out of their former place to another place further on in the channel, and they repose in their new place at rest as at first. Here is a final effect, and here my agency has ceased—not so the motion of the water; I pushed forward a given mass out of its place into another, but that other place was formerly occupied by a mass of water equal to that which I have forcibly intruded into its place; what then has become of the displaced occupant? it has been forced into the place of that immediately before it, and the occupant which it has dislodged is again pushed forward on the occupant of the next place, and thus in succession volume after volume continues to carry on a process of displacement which only ends with the termination of the channel, or with the exhaustion of the displacing force originally impressed by my hand, and communicated from one to another successive mass of the water. This process continues without the continuance of the original disturbing agency, and is prolonged often to great distances and through long periods of time. The continuation of this motion is therefore independent of the volition which caused it. It is a process carried on by the particles of water themselves obeying two forces, the original force of disturbance and the force of gravity. It is therefore a hydrodynamical phenomenon conformable to fixed law. I have now ceased to exercise any control over the phenomenon, but as I attentively watch the processes I have set a-going, I observe each successive portion of water in the act of being displaced by one moving mass of water, and in the act of displacing its successor. As the water particles crowd upon one another in the act of going out of their old places into the new, the crowd forms a temporary heap visible on the surface of the fluid, and as each successive mass is displacing its successor, there is always one such heap, and this heap travels apparently along the channel at that point where the process of displacement is going on, and although there may be only one crowd, yet it consists successively of always another and another set of migrating particles.

This *visible moving heap of crowding particles* is a true *wave*, the rapidity with which the displacement of one outgoing mass by that which takes its place, goes forward, determines the velocity with which the heap appears to move, and is called the *velocity of transmission* of the wave. The shape which the crowding of the particles gives to the surface of the water constitutes the *form of the wave*. The distance (in the direction of the transmission) along which the crowd extends, is called the *length or amplitude* of the wave. The number of particles which at any one time are out of their place, constitute the *volume of the wave;* the time which must elapse before particles can effect their translation from their old places to the new, may be termed the *period of the wave.* The *height of the wave* is to be reckoned from the highest point or crest to the surface of the fluid when in repose.

Such is the wave motion—very different is the *water motion.* Let us select from the crowd of water particles an individual and watch its behaviour during the migration. The progressive agitation first reaches it while still in perfect repose; the crowd behind it push it forward and new particles take its place. One particle is urged forward on that before it, and being still urged on from behind by the crowd still swelling and increasing, it is *raised* out of its place and *carried forward* with the velocity of the surrounding particles; it is urged still on until the particles which displaced it have made room for themselves behind it, and then the power diminishes. Having now in its turn pushed the particles before it along out of their place, and crowded them together on their antecedents, it is gradually left behind and finally *settles quietly down in its new place.* Thus then the *motion of migration of an individual particle* of water is very different from the *motion of transmission of the wave.*

The wave goes still forward along through the channel, but each individual water particle remains behind. The wave passes on with a continuous uninterrupted motion. The water particle is at rest, starts, rises, is accelerated, is slowly retarded, and finally stops still. The *range of the particle's motion* is short; its *translation* is interrupted and final. Its *vertical range* and *horizontal range are finite*. It describes an *orbit* or path during the *transit of the wave* over it, and remains for ever after at rest, unless when a second wave happens immediately to follow the first, when it will describe a second time *its path of translation*, passing through a series of new positions or *phases* during the *period of the wave*. The motion of the particle is not therefore like the apparent motion of the wave, either uniform or continuous. The motion of the water particles is a true motion of translation of matter from one place to another, with the velocity and range which the senses observe. But the wave motion is an ideal individuality attributed by the mind of the observer to a process of changes of relative position or of absolute place, which at no two instants belongs to the same particles in the same place. The water does not travel, the visible heap at no two successive instants is the same. It is the motion of particles which goes on, now at this place, now at that, having passed all the intermediate points. *It is the crowding motion alone which is transmitted.* This crowding motion transmitted along the water idealised and individualised is a true wave.

Wave propagation therefore consists in the transmission from one class of particles to another, of a motion differing in kind from the motion of transmission. *Wave motion* is therefore transcendental motion; motion in the second degree; the motion of motion—the transference of motion without the transference of the matter, of form without the substance, of force without the agent.

It is essential to the accurate conception and examination of waves, that this *distinction between the wave motion and the water motion* be clearly conceived. It has been well illustrated by the agitations of a crowd of people, and of a field of standing corn waving with the wind. If we stand on an eminence, we notice that each gust as it passes along the field bending and crowding the stalks, marks its course by the motion it gives to the grain, and the visible effect is like that of an agitated sea.

In the examination of the phenomena of waves, we have therefore two classes of elements for consideration, the elements of the wave motion and the elements of the water particle motion. We may first examine the *phenomena of a given wave-motion,* its range of transmission over the surface of the fluid, the velocity of that transmission, the form of the elevation, its amplitude, breadth, height, volume, period. We may next consider the *path which each water particle describes* during the wave transit; the *form* of that path, the *horizontal or vertical range* of the motion, the variation of the path with the depth, the relation of each *phase of the particle's orbit* to each portion of the corresponding wave length. By this examination I have found that there exist among waves groups of phenomena so different as to suggest their division into *distinct classes.* I find that the general form of waves is manifestly different, one kind of wave making its appearance in a form always wholly raised above the general surface of the fluid, and which we may call a *positive wave,* and so distinguish it from another form of wave which is wholly *negative,* or depressed below the plane of repose, while a third class are found to consist of both a negative and a positive portion. I find them propagated with extremely different velocities, and obeying different laws according as they belong to one or the other of these classes, the positive wave having *in a given depth of water a constant and invariable velocity*, while another class has a *velocity varying* according to other peculiarities, and *independent of the depth.* Some of them again are distinguished by always appearing alone as individual waves, and others as *companion phenomena* or *gregarious*, never appearing except in groups. In examining the paths of the water particles corresponding differences are observed. In some the water particles perform a *motion of translation* from one place to another, and effect a permanent and final change of place, while others merely change their place for an instant to resume it again; thus performing *oscillations* round their place of final repose. These waves may also be distinguished by the sources from which they arise, and the forces by which they are transmitted. One class of wave is a *motion of successive transference* of the whole fluid mass; a second, the *partial oscillation* of one part of it without affecting the remainder; a third, the propagation of an impulse by the *corpuscular forces* which determine the elasticity of the fluid mass; and a fourth, by the *capillary forces* uniting its molecules at the surface.

TABLE I.

System of Water Waves.

ORDERS.	FIRST.	SECOND.	THIRD.	FOURTH.
Designation.	Wave of translation............	Oscillating waves.	Capillary waves.	Corpuscular wave.
Character....	Solitary	Gregarious.........	Gregarious.......	Solitary.
Species	Positive..........	Stationary.........	Free.	
	Negative	Progressive.........	Forced.	
Varieties	Free..............	Free.		
	Forced...........	Forced.		
Instances	The wave of resistance...	Stream ripple.....	Dentate waves	Water-sound wave.
	The tide wave	Wind waves.......	Zephyral waves.	
	The aërial sound wave	Ocean swell.......		

An observer of natural phenomena who will study the surface of a sea or large lake during the successive stages of an increasing wind, from a calm to a storm, will find in the whole motions of the surface of the fluid, appearances which illustrate the nature of the various classes of waves contained in Table I., and which exhibit the laws to which these waves are subject. Let him begin his observations in a perfect calm, when the surface of the water is smooth, and reflects like a mirror the images of surrounding objects. This appearance will not be affected by even a slight motion of the air, and a velocity of less than half a mile an hour ($8\frac{1}{2}$ in. per sec.) does not sensibly disturb the smoothness of the reflecting surface. A gentle zephyr flitting along the surface from point to point, may be observed to destroy the perfection of the mirror for a moment, and on departing, the surface remains polished as before; if the air have a velocity of about a mile an hour, the surface of the water becomes less capable of distinct reflection, and on observing it in such a condition, it is to be noticed that the diminution of this reflecting power is owing to the presence of those minute corrugations of the superficial film which form waves of the *third order*. These corrugations produce on the surface of the water an effect very similar to the effect of those panes of glass which we see corrugated for the purpose of destroying their transparency, and these corrugations at once prevent the eye from distinguishing forms at a considerable depth, and diminish the perfection of forms reflected in the water. To fly-fishers this appearance is well known as diminishing the facility with which the fish see their captors. This first stage of disturbance has this distinguishing circumstance, that the phenomena on the surface cease almost simultaneously with the intermission of the disturbing cause, so that a spot which is sheltered from the direct action of the wind remains smooth, the waves of the third order being incapable of travelling spontaneously to any considerable distance, except when under the continued action of the original disturbing force. This condition is the indication of present force, not of that which is past. While it remains it gives that deep blackness to the water which the sailor is accustomed to regard as an index of the presence of wind, and often as the forerunner of more.

The second condition of wave motion is to be observed when the velocity of the wind acting on the smooth water has increased to two miles an hour. Small waves then begin to rise uniformly over the whole surface of the water; these are waves of the second order, and cover the water with considerable regularity. Capillary waves disappear from the ridges of these waves, but are to be found sheltered in the hollows between them, and on the anterior slopes of these waves. The regularity of the distribution of these secondary waves over the surface is remarkable; they begin with about an inch of amplitude, and a couple of inches long; they enlarge as the velocity or duration of the wave increases; by-and-by conterminal waves unite; the ridges increase, and if the wind increase the waves become cusped, and are regular waves of the *second order.* They continue enlarging their dimensions, and the depth to which they produce the agitation increasing simultaneously with their magnitude, the surface becomes extensively covered with waves of nearly uniform magnitude.

How it is that waves of unequal magnitude should ever be produced may not seem at first sight very obvious, if all parts of the original surface continue equally exposed to an equal wind. But it is to be observed that it rarely occurs that the water is all equally exposed to equal winds. The configuration of the land is alone sufficient to cause local inequalities in the strength of the wind and partial variations of direction. The transmission of reflected waves over such as are directly generated by the wind, produces new forms and inequalities, which, exposed to the wind, generate new modifications of its force, and of course, in their turn, give rise to further deviations from the primitive condition of the fluid. There are on the sea frequently three or four series of co-existing waves, each series having a different direction from the other, and the individual waves of each series remaining parallel to one another. Thus do the condition, origin, and relations of the waves which cover the surface of the sea after a considerable time, become more complex than at their first genesis.

It is not until the waves of the sea encounter a shallow shelving coast, that they present any of the phenomena of the wave of the first order. After breaking on the margin of the shoal, they continue to roll along in the shallow water towards the beach, and becoming transformed into waves of the first order, finally break on the shore.

But the great example of a wave of the *first order*, is that enormous wave of water which rolls along our shores, bringing the elevation of high tide twice a day to our coasts, our harbours, and inland rivers. This great compound wave of the first order is not the less real that its length is so great, that while one end touches Aberdeen, the other reaches to the mouth of the Thames and the coast of Holland. Though the magnitude of this wave renders it impossible for the human eye to take in its form and dimensions at one view, we are able, by stationing numerous observers along different parts of the coasts, to compare its dimensions and to trace its progress at different points, and so to represent its phenomena to the eye and the mind on a small scale, as to comprehend its form and nature as clearly as we do those of a mountain range, or extensive country which has been mapped on a sheet of paper by the combination together of trigonometrical processes, performed at different places by various observers, and finally brought together and protracted on one sheet of paper.

I believe I shall best introduce this phenomenon by describing the circumstances of my own first acquaintance with it. I was observing the motion of a boat which was rapidly drawn along a narrow channel by a pair of horses, when the boat suddenly stopped—not so the mass of water in the channel which it had put in motion; it accumulated round the prow of the vessel in a state of violent agitation, then suddenly leaving it behind, rolled forward with great velocity, assuming the form of a large solitary elevation, a rounded, smooth, and well-defined heap of water, which continued its course along the channel apparently without change of form or diminution of speed. I followed it on horseback, and overtook it still rolling on at a rate of some eight or nine miles an hour, preserving its original figure some thirty feet long and a foot to a foot and a half in height. Its height gradually diminished, and after a chase of one or two miles I lost it in the windings of the channel. Such, in the month of August 1834, was my first chance interview with that singular and beautiful phenomenon which I have called the Wave of Translation, a name which it now very generally bears; which I have since found to be an important element in almost every case of fluid resistance, and ascertained to be the type of that great moving elevation of the sea, which, with the regularity of a planet, ascends our rivers and rolls along our shores.

To study minutely this phenomenon with a view to determine accurately its nature and laws, I have adopted other more convenient modes of producing it than that which I have just described, and have employed various methods of observation. A description of these will probably assist me in conveying just conceptions of the nature of this wave.

Genesis of the Wave of the First Order.

For producing waves of the first order on a small scale, I have found the following method sufficiently convenient. A long narrow channel or box a foot wide, eight or nine inches deep, and twenty or thirty feet long (Plate I. fig. 1), is filled with water to the height of say four inches. A flat board P (or plate of glass) is provided, which fits the inside of the channel so as to form a division across the channel where it is inserted.

Genesis by Impulsion or Force horizontally applied.

Let this plate be inserted vertically in the water close to the end A, and being held in the vertical position, be pressed forward slowly in the direction of X, care being taken that it is kept vertical and parallel to the end. The water now displaced by the plate P in its new position accumulates on the front of the plane forming a heap, which is kept there, being enclosed between the sides of the channel and the impelling plate. The amount thus heaped up is plainly the volume of water which has been removed by the advancing plane from the space left vacant behind it, and if the impulse increase, the elevation of displaced water will increase in the same quantity. When the water has reached the height P_3, let the velocity of impulsion be now gradually diminished as at P_4, until the plate is finally brought to rest as at

P_5; the height of the water heaped on the front will diminish with the diminution of velocity as at P_4, and when brought to rest at P_5 it will be on the original level. The total height of the water does not, however, subside with the diminution of the impulsion, the crest W_4 retains the maximum height to which it had risen under the pressure of the plane at P_3, and moves horizontally forward; and the smaller elevation produced by the smaller pressure at P_4 down to P_5 moves forward after W_5. This elevation of the liquid, having a *crest*, or summit, or *ridge* in the centre of its length transverse to the side of the channel, continues to move along the channel in the direction of the original impulsion; from the crest there extends forward a curved surface, Wa, forming the *face* of the *wave*, and a similar surface, Ww, behind the crest, is distinguished as its *back*. It is convenient to designate a as the *origin*, w as the *end* of the wave; and to designate the interval between a and w, the length of the wave in the direction of its transmission, its *amplitude*.

The kind of motion required for generating this wave in the most perfect way, that is, for producing a wave of given magnitude without at the same time creating any disturbance of a different kind in the water—this kind of motion may be given by various mechanical contrivances, but I have found that the dexterity of manipulation which experience bestows is perfectly sufficient for ordinary experimental purposes.

Genesis by a Column of Fluid.

This is a method of genesis of considerable value for various experimental purposes, especially useful when waves of no great magnitude are required, and also when it is desirable to measure accurately volumes or forces employed in wave genesis. The same glass plate may be conveniently employed in this case as in the last, only it will now be used in the capacity merely of a sluice, and be supported by two small vertical slips fixed to the sides of the channel so as to keep it in the vertical position, but to admit of its being raised vertically upwards as at G, Pl. I. fig. 2. There is thus formed between the end of the channel G and the movable plate P_7, a small generating reservoir GP_7. This is to be filled to any desired height with water, as from w to P_7, and the plate being drawn up, as at P_9, the water of the reservoir descends to w, the level of the water of the channel, and pushing forward and heaping up the adjacent fluid, raises a heap equal to the added volume on the surface of the water; and this elevation is in no respect sensibly different either in form or other phenomenon from that generated in the former method, provided the quantity of water added in the latter case be identical with the quantity of water displaced in the former case.

This method of genesis by fluid column affords a simple means of proving an elementary fact in this kind of wave motion. The fact is this, that while the volume of water in the wave is exactly equal to the volume of water added from the reservoir, it is by no means identical with it. I filled the reservoir with water tinged with a pink dye, which did not sensibly alter the specific gravity of the water. The column of water having descended as at K, and the wave having gone forward to W_9, the generating column remained stationary at K, thus indicating that the column of water had merely acted as a mechanical prime mover, to put in action the wave-propagating forces among the fluid, in the same way as had been formerly done by the power acting by the solid plate in the former case of genesis by impulsion. Thus is obtained a first indication that this wave exhibits *a transmission of force, not of fluid,* along the channel.

Genesis by Protrusion of a Solid.

The quantity of moving force required for the wave-genesis may be directly obtained by the descent of a solid weight. The solid at L (fig. 3) may be a box of wood or iron, containing such weights as are desired, and suspended in such a manner as to be readily detached from its support. Its under surface should be somewhat immerged. On touching the detaching spring, the weight descends, and the water it displaces produces a wave of equal volume. If the weight and volume of solid thus immersed be equal to those of the water in the reservoir in the former case, it is found that the waves generated by the two methods are alike. It is expedient that the breadth and shape of this solid generator should be such as to fit the channel, as this removes some sources of disturbance. The results which are produced by this application of moving power are also convenient for giving measures of the mechanical forces employed in wave-genesis.

This method is especially convenient for the genesis of waves of considerable magnitude. With this view

I erected a pyramidal structure of wood, capable of raising weights of several hundred pounds, over a pulley by means of a crane, and contrived to allow them to descend at will. This apparatus was adequate for the generation of waves in a channel three feet wide and three feet deep; and the same construction may be extended to greater dimensions.

Transmission of Mechanical Power by the Wave.

By the last two methods of genesis there is to be obtained a just notion of the nature of the wave of the first order as a vehicle for the transfer of mechanical power. By the agency of this wave the mechanical power which is employed in wave-genesis at one end of the channel, passes along the channel in the wave itself, and is given out at the other end with only such loss as results from the friction of the fluid. At one end, as of the channel G, fig. 4, there is placed the water, which, falling through a given height, is to generate the wave. At the other end, X, is a similar reservoir and sluice, open to the channel. When the wave has been generated as at K, and has traversed the length of the channel, it enters the receptacle X, and assuming the form marked at L, the sluice being suddenly permitted to descend, the column of water will be enclosed in the receptacle, and its whole volume raised above the level of repose nearly as at the first. The power expended in wave-genesis, having been transferred along the whole channel, is thus once more stored up in the reservoir at the other extremity. A part of this power is, however, expended *in transitu* by friction of the particles and imperfect fluidity, &c. When the channel is large, the sides and bottom smooth, the transmission of force may be accomplished with high velocity, at the rate of many miles an hour, to a distance of several miles.

Re-genesis of Wave.

In the channel AX, we have found the wave transmitted from A to X, and there the power of genesis transferred to the fluid column now stored up in the reservoir X. If we now repeat from the receptacle X the same process of genesis originally performed at G, elevating the sluice and allowing the fluid column to descend, it will again generate a wave similar to the first, only transmitted back in the opposite direction. This re-generated and re-transmitted wave may be again found in the primary reservoir of genesis as at G, and the same power, after having been transmitted twice through the length of the channel, be restored as at first in that channel, with only the small diminution of power lost *in transitu.* The process of re-genesis may now be repeated, as at first, and so on during any number of successive transmissions and re-transmissions.

Reflection of the Wave.

This process of restoring the force employed in wave-genesis, and of re-genesis of the wave, may take place without the intervention of the sluices. The wave, on reaching the end of the channel G at X_7, becomes accumulated in the form of the curve

$w\,x$. We have therefore the power of genesis now stored up in this water column, $w\,L\,x$, above the level L, and in a state of rest. By means of a sluice we may detain it at that height for as long time as we please. But let us suppose we do not wish to detain it, but allow the water column to descend by gravity as at first, it generates the wave by again descending, and transmits it back towards G, as effectually as if the reservoir had been used, or as the genesis when first accomplished. By the same process of *laissez faire*, the power of genesis will be restored at G, a water column elevated, the fluid brought to rest and allowed again to descend, again to effect genesis of the wave, and again transmit the force along the channel through the particles of the wave. The wave is said to be reflected, and it is thus shown in reference to the wave of the first order, that the process called reflection consists in a process of restoration of the power of genesis, and of regenesis of the wave in an opposite direction. In this manner there is to be obtained an accurate view of the mechanical nature of the reflection of the wave.

Many other modes of genesis have been employed; solids elevated from the bottom of the channel, vessels drawn along the channel, &c.; wherever a considerable addition is made to the height and volume of the liquid at any given point in the channel, a wave of the first order is generated, differing in no way from the former, except in such particulars as are hereinafter noticed.

Motion of Transmission.

The crest of the wave is observed to move along a channel which does not vary in dimension, with a *velocity sensibly uniform*, so that the velocity with which it is transmitted may be determined by simply measuring a given distance along the channel, and observing the number of seconds which may elapse during the transit from one end of the line to the other. This interval of time is sensibly equal for any equal space measured along the path, and hence we determine that the velocity of the wave transmission is sensibly uniform.

Range of Wave Transmission.

The distance through which a wave of the first order will continue to propagate itself is so great as to afford considerable facility for accurate observation of its velocity. For accurate observations it is convenient to allow the early part of the range to escape without observation; for this purpose, that the primary wave, which is to be the subject of observation, may disèmbarrass itself of such secondary phenomena as frequently accompany its genesis, when that genesis cannot be accurately accomplished. A small part of the range is sufficient for this purpose, and the remainder is perfectly adapted for purposes of accurate observation, as it continues to travel along its path long after the secondary waves have ceased to exist. The *longevity of the wave of the first order*, and the facility of observing it, may be judged of from the following experiments, made in 1835–1837.

Ex. 1. A wave of the first order, only 6 inches high at the crest, had traversed a distance of 500 feet, when it was first made the subject of observation. After being transmitted along a further distance of 700 feet, another observation was noted, and it was observed still to have a height of 5 inches, and to have travelled with a velocity of 7.55 miles an hour.

Ex. 2. A wave of the first order, originally 6 inches high, was transmitted through a distance of 3200 feet, with a mean velocity of 7.4 miles an hour, and at the end of this path still maintained a height of 2 inches.

Ex. 3. A wave 18 inches high, moving at the rate of 15 miles an hour, in a channel 15 feet deep, had still a height of 6 inches, having traversed the same space in 12 minutes.

Ex. 4. Among small experimental waves of the first order, in small channels, I have selected one, whose crest being 1.34 inch high, in a channel 5.10 inches deep, was transmitted through a range of 1360 feet, and still admitted of accurate observation.

These examples serve to convey an accurate idea of the longevity of a wave of the first order. And this longevity appears to increase with the depth and the breadth of the channel, and with the height of the wave crest.

Degradation of the Wave of the First Order.

In the progress of a wave of the first order, it is observed that its height diminishes with the length of its path; the velocity also diminishes with the diminution of height, though very slowly. This degradation of height is observed to go on more rapidly in proportion as the channel is narrow, shallow, or irregular, and rough on the sides, and is diminished according as the channel is made smooth and regular in its form, or deep and wide. It is to be attributed to the imperfect fluidity of the water in some degree, but also to the adhesion of water to the sides. The particles of fluid near the sides and bottom are retarded in their motions, and the transmission takes place more slowly among them. The wave passes on, leaving in these particles a small quantity of the motion it had communicated, and of its force and volume, and in consequence of this there exists along the whole channel, over which the wave has passed, a residual motion, or continuous residual wave, very small in amount, but still appreciable by accurate means of observation. The volume of the wave is thus diffused over a large extent along its path, where finally it has deposited the whole of its volume, and so disappears. This degradation is therefore the means by which the motion of a wave in an indefinite channel is gradually and slowly terminated. In the history of a solitary wave of the first order, the progress of this degradation is to be observed from the examination of Table II. column B, which gives the height of the wave as observed at every 40 feet along its path. In the first 200 feet this diminution amounts to about $\frac{1}{4}$ of the height at the commencement. At the end of the second 200 feet, the height is diminished by $\frac{4}{9}$ of the height at the commencement of that space. During the third space of 200 feet the degradation produced is nearly $\frac{1}{2}$ of the height of the wave. This appears to be the most rapid degradation, and in the next space of 200 feet it is little more than $\frac{1}{3}$; in the next, less than a third of the height at the beginning of that space. These successive heights are given graphically in Plate II. fig. 7.

The Velocity of Transmission of the Wave of the First Order.

The history of a single wave has sufficed to show us that the velocity with which its crest is transmitted along the channel is nearly that which a heavy body will acquire falling freely through a height equal to half the depth of the fluid. This is a very simple and important character in the phenomena of this wave, by which, when the depth of the channel is known, we may at once predict approximately the velocity of the wave of translation. The following are approximate numbers deduced from this conclusion, and which I find it convenient to recollect.

In a channel whose depth is $2\frac{1}{2}$ inches, the velocity of the wave is $2\frac{1}{2}$ feet per second.

In a channel whose depth is 15 feet, the velocity of the wave is 15 miles an hour.

In a channel whose depth is 90 fathoms, the velocity of the wave is 90 miles an hour.

These numbers are, however, only first approximations, for it is to be observed that the wave, when its height is considerable, moves with greater velocity than when it is small. These numbers become accurate, if in the depth, the height of the wave be included.

The Height of the Wave of the First Order, an Element in its Velocity.

The height of the wave appears to enter as an element in its velocity, and to cause it to deviate from the simple formula (A). Thus the velocity of the wave only coincides with the velocity assigned in Table II. (Appendix, p. 226), when the height of the wave is inconsiderable.

I have found that this deviation is to be reconciled, without at all destroying the simplicity of the formula, by a very simple means. In order to obtain perfect accuracy, we have only to reckon the effective depth for calculation, from the ridge or crest of the wave instead of from the level of the water at rest; and having thus added to the depth of the water in repose, the height of the wave crest over the plain of repose, if we take the velocity which a heavy body would acquire in falling through a space equal to half the depth of the fluid (reckoning from the ridge of the wave to the bottom of the channel), that number accurately represents the velocity of transmission of the wave of the first order.

We have, therefore, for the velocity of the wave of the first order,

approximately $v=\sqrt{gh}$, ... (A)

accurately $v=\sqrt{g\,(h+k)}$, ... (B)

where v is the velocity of transmission,

g is the force of gravity as measured by the velocity which it will communicate in a second to a body falling freely $= 32$,

h is the depth of the fluid in repose,

k is the height of the crest of the wave above the plane of repose.

The velocities of waves of the first order in channels of different depths are, therefore, as the square roots of the depth of these channels.

Nevertheless, when the height of one of the waves is considerable compared with the depth of the channel, a high wave in the shallower channel may move faster than a lower wave in a deeper channel; provided only the excess in height of the higher wave be greater than the difference of depth of the channels; in short, that wave will move fastest in a given channel whose crest is highest above the bottom of the channel, and in channels of different depths waves may be propagated with equal velocities, provided only the sum of the height of wave and standing depth of channel amount to the same quantity.

Wave of the First Order not formerly described.

Although many distinguished philosophers from the time of Sir Isaac Newton have devoted themselves to the study of the theory of waves, I have not been able to discover in their works anything like the prediction of a phenomenon such as the wave of translation or the solitary wave of the first order. The waves of the second order, or gregarious oscillations, which make their appearance in successive groups, or long and recurring series, such oscillations of the surface of the water as we notice on the sea, or are excited when the quiescent surface of a lake is disturbed by dropping a stone, and which diffuse themselves in concentric circles around the centre of derangement; these have long been familiar to naturalists, and have been studied, though with comparatively little success, by philosophers. But I have not found the phenomenon, which I have called the wave of the first order, or the great solitary wave of translation, described in any observations, nor predicted in any theory of hydrodynamics.

The wave of the first order bears as its characteristics, the observed phenomena, that the agitation does extend below the surface to the very bottom of the channel, where it is quite as great as at the surface, and that its oscillations are large.

The Magnitude and Form of the Wave of the First Order.

The exact determination of the dimensions and form of the wave, although at first sight it may seem simple enough, is not without peculiar difficulties. When it is observed that the two extremities of the wave are vertices of curves of very small curvature tangent to the plane of repose, it will be understood how difficult it is to detect the place of contact with precision. A variety of methods have been tried: reflection of an image from the surface, tangent points applied to the surface so as to be observed simultaneously at both ends of the wave, and the self-registration of a float moved by the wave, have all been tried with various success. On the whole, however, the most perfect observations have been obtained by a very simple autographic method, in which it was contrived that the wave should leave its own outline delineated on the surface without the intervention of any mechanism.* The method was simply this:

* I find that I am not the first person who employed an apparatus of this sort. MM. Webers employed a powdered surface to register the form of agitated mercury, the fluid rubbing off the powder.

a dry smooth surface was placed over the surface of the water in the channel, with such arrangements that it could be moved along with the velocity of transmission of the wave, and at the instant of observation it was pushed vertically down on the wave, and raised out again without sensibly disturbing the water; the surface when brought out, brought with it a moist outline of the wave, which was immediately traced by pencil, and afterwards transferred to paper.

Another method of obtaining an autographic representation of waves of the first order was this. Two waves were generated at opposite ends of the same channel at given instants of time, so that by calculating their velocities they should both reach a given spot at the same instant; here a prepared surface was placed, and as one passed over the other it left a beautiful outline of the excess in height of each point of one wave above the summit height of the other. These forms are not identical with those of the same wave moving along a plane surface, but as true registers of actual phenomena they are interesting.

The results of all my observations on this subject are as follows:—

That the wave of the first order has a definite *form and magnitude* as much characteristic of it as the uniform velocity with which it moves, and depending like that velocity only on the depth of the fluid and the height of the wave crest.

That this wave-form has its surface wholly raised above the level of repose of the fluid. This is what I mean to express by calling this wave *wholly positive.* I apply the word negative to another kind of wave whose surface exhibits a depression below the surface of repose. The wave-proper of the first order is wholly positive.

The simple elementary wave of the first order assumes a definite *length equal to about six times the depth of the fluid below the plane of repose.* When the height of the wave is small the length does not sensibly differ from that of the circumference of a circle whose radius is the depth of the fluid.

The height of the wave above the surface of the water in repose may increase till it be equal to the depth of the fluid in repose. When it approaches this height it becomes acuminate, finally cusped, and falls over, breaking and foaming with a white crest. The limits of the wave height are, therefore,

$$k = 0, \text{ and } k = h;$$

that is to say, the height of the wave may increase from O to K.

The methods I had employed for such observations as I had then already made, were the observation of the motions of small particles visible in the water of the same, or nearly the same specific gravity with water, or small globules of wax connected to very slender stems, so as to float at required depths. The motions of these were observed from above, on a minutely divided surface on the bottom of the channel, and from the side through glass windows, themselves accurately graduated, the side of the channel opposite to the window being covered with lines at distances precisely equal to those on the window and similarly situated. These methods are the only methods of observation I have found it useful to employ, but I have now increased the number and variety of the observations sufficiently to enable me to adduce the conclusions hereinafter following, as representing the phenomena as far as their nature will admit of accurate observation.

It is characteristic of waves that the *apparent motion visible on the surface* of the water is of one species, while the *absolute motion of the individual particles* of the water is very different. In reference to all the species of waves this is true, both as regards the velocity and nature of the motion; nevertheless the one is the immediate cause or consequence of the other. In the case of the wave of the first order, the visible motion of the wave form along the surface of the water may be called the *motion of transmission;* the actual motion of the particles themselves is to be distinguished as the *motion of translation.*

We infer the motions of the individual wave particles from those of visible small bodies floating in the water; any minute particle floating on the surface will sufficiently indicate the motion of the water particles about it, and the motion of deeper particles may be conveniently observed in the case of waves of the first order, by using the little globules of wax already mentioned; these small globules may be so made as to float permanently at any given depth, yet they will be visibly affected by very minute forces.

In this way the following observations were made:

Absolute Motion of Translation.

The phenomenon of translation characteristic of the wave of the first order, and which we have used as its distinguishing appellation, is to be observed as follows. Floating globules being placed in the fluid, and their positions being noted with reference to the sides and bottom of the channel, let a wave of the first order be transmitted along the fluid; it is found that the effect of this transmission is to lift each of the floating particles, and similarly, therefore, the water particles themselves, out of their positions, and to transfer them permanently forward to new positions in the channel, and in these new positions the particles are left perfectly at rest, as in their original places in the channel.

The measure or range of translation is just equal to that which would result from increasing the column of water in the channel behind the wave by a given quantity, and diminishing the column anterior to the particles by the same quantity, that quantity being equal to the volume of the wave. That is to say, *the range of translation is simply equal to the space in length of the channel which the volume of the wave would occupy* on the level of the water in repose.

The *total effect* of having transmitted a wave of the first order along a channel, is to have moved successively every particle in the whole channel forward, through a space equal to the volume of the wave divided by the water-way of the channel.

Parallelism of Translation.

If the floating spherules before-mentioned be arranged in repose in one vertical plane at right angles to the direction of transmission, and carefully observed during transmission, it will be noticed that the particles remain in the same plane during transmission and repose in the same place after transmission.

It is further found, as might be anticipated from the foregoing observations, that a thin solid plane transverse to the direction of transmission, and so poised as to float in that position, does not sensibly interfere with the motion of translation or of transmission.

The Range of Horizontal Translation is equal at all depths.

Vertical excursions are performed by each particle of fluid simultaneously with the horizontal translation. These diminish in extent with the distance from the bottom when they become zero.

The Path of each Water Particle during Translation lies wholly in a Vertical Plane.

It may be observed by means of the glass windows already mentioned, its surface being graduated for purposes of measurement. The path is so rapidly described that I do not think any measurements of time which I have made, nor even of paths, is *minutely* correct. The following observations are such as a practised eye with long experience and much pains has made out.

When a wave of the first order in transmission makes a transit over floating particles in a given transverse plane, the observations are as follows:— All the particles begin to rise, scarcely advancing; they next advance as well as rise; they cease to rise but continue advancing; they are retarded and come to rest, descending to their original level. The path appears to be an ellipse whose major axis is horizontal and equal to the range of translation; the semi-minor axis of the elliptic path is equal to the height of the wave near the surface, and diminishes directly with the depth.

Mechanism of the Wave.

The study of the phenomena of the translation of water particles during the transit of a wave is peculiarly valuable, as affording us the means of correctly conceiving the real nature of wave transmission of the first order; it therefore deserves great attention.

We perceive, in the first place, that the vertical arrangement of the water particles is not deranged by wave transmission; that is, if we conceive the whole fluid in repose to be intersected by transverse vertical planes, thin, and of the specific gravity of water, these planes will retain their parallelism during transmission, and will not affect that transmission.

The power employed in wave genesis is therefore expended in raising to a height, equal to the crest of the wave, each successive water column; each water column, again descending, gives out that measure of power to the next in succession, which it thus raises to its own height. The time employed in raising a given column to this height, and in its descent and communication of its own motion to the next in succession, constitutes the period of a wave, and the number of such columns undergoing different stages of the process at the same time measures the length of a wave.

During the anterior half of the wave the following processes take place :—The generating force communicates to the adjacent column through its posterior bounding plane, a pressure; this pressure moves the posterior plane forward, the water in the column is thereby raised to the height due to the velocity, and the pressure of this water column communicates to the anterior bounding plane also a velocity and a pressure in the same direction; therefore the accelerating force produces a given motion of translation in the whole column, a height of column due to that velocity, and an approximation of the anterior forces of the column to each other; these are all the forces and the motions concerned in the matter. The motive power thus stored during the anterior half of the wave is restored in the latter half wave length thus; the column raised to its greatest height presses on both its posterior and anterior surface, on the anterior surface it presses forward the anterior column, tending to sustain its velocity and maintain its height; on the posterior column its pressure tends to oppose the progress and retard the velocity of the fluid in motion, and thus retarding the posterior and accelerating the anterior surface, widens the space between its own bounding planes until it reposes once more on the original level.

The Wave a Vehicle of Power.

The wave is thus a receptacle of moving power, of the power required to raise a given volume of water from its place in the channel to its place in the wave, and is ready to transmit that power through any distance along that channel with great velocity, and to replace it at the end of its path. In doing this the motion of the water is simple and easily understood, each column is diminished in horizontal dimension and increased proportionally in vertical dimension, and again suffered to regain its original shape by the action of gravity. There is no transference of individual particles through, between and amongst one another, so as to produce collisions, or any other motions which impair moving force; the particles simply glide for the moment over each other into a new arrangement, and retire back to their places. Thus the wave resembles that which we may conceive to pass along an elastic column, each slice of which is squeezed into a thinner slice, and restored by its elastic force to its original bulk, only in the water wave the force which restores the force of each water column is gravity, not elasticity.

To conceive accurately of the forces which operate in wave transmission, and of the *modus operandi*, to understand how the primary moving force acts on the column of fluid in repose, how this force is distributed among the particles, to distinguish the relative and absolute motions of the particles and the nature of the transmission of the form, and to understand how the force operates in at once propagating itself and restoring completely to rest those particles which form the vehicle of its transmission, is a study of much interest to the philosopher. To show how under a given form and outline of wave, in a given time, all and each of the individual particles of water obeying every one its own impulse and that of those around it, and subject to the laws of gravity and of the original impulse, shall describe its own path without interfering with another's, and shall unite in the production of an aggregate motion consistent with the continuity of the mass and with the laws of fluid pressure,—this is a problem which belongs to the mathematician, which has hitherto proved too arduous for the human intellect, and which we have thus endeavoured to facilitate and promote by the study of the absolute forms and phenomena of the waves themselves, and by the determination of the actual paths and motions of the individual particles of water.

On some Conditions which affect the Phenomena of the Wave of the First Order.

It has not appeared in any observations I have been able to make on the subject, that the wave of the first order retains the stamp of the many peculiarities that may be conceived to affect its origin. In this respect it is apparently different from the waves of sound or of colour, which bear to the ear and the eye distinct indications of many peculiarities of their original exciting cause, and thus enable us to judge of the character of the distant cause which emitted the sound or sent forth the coloured ray. It is not possible always to form an accurate judgment from the phenomena of the wave of the first order, of the nature of the disturbing cause, except in peculiar and small number of cases.

I have not found that waves generated by impulse, by a fluid column of given and very various dimension, by immersion of a solid body of given figure, by motion in given velocity or in different directions; I have not found in the wave obtained by any of these many means any peculiarity, any variation either of form or velocity, indicating the peculiarity of the original. In one respect, therefore, the wave of translation resembles the sound wave; that all waves travel with the velocity due to half the depth, whatever be the nature of their source.

In one respect alone does the origin of the wave affect its history. Its volume depends on the quantity of power employed in its genesis, and on the distance through which it has travelled. A great and a little wave at equal distances from the source of disturbance, arise from great or little causes, but it is impossible to distinguish between a small wave which has travelled a short distance, and one which, originally high, has traversed a long space.

Form of Channel—Its Effect on the Wave of Translation.

The conditions which affect the phenomena of the wave of translation are therefore to be looked for in its actual circumstances at the time of observation rather than in its history. The form and magnitude of the channel are among the most important of these circumstances. Thus a change in depth of channel immediately becomes indicated to the eye of the observer by the retardation of the wave, which begins to move with the same velocity as if the channel were everywhere of the diminished depth, that is, with the velocity due to the depth. Thus in a rectangular channel $4\frac{1}{2}$ feet deep, the wave moves with a velocity of 12 feet per second, and if the channel becomes shallower, so as to have only two feet depth, the change of depth is indicated by the velocity of the wave, which is observed now to move only with the velocity of 8 feet per second; but if the channel again change and become 8 feet deep, the wave indicates the change by suddenly changing to a velocity of 16 feet per second.

Length of Wave an Index of Depth.

In like manner, a wave which in water 4 feet deep is about 8 yards long, shortens on coming to a depth of 2 feet to a length of 4 yards, and extends itself to 16 yards long on getting into a depth of 8 feet. This extension of length is attended with a diminution of height, and the diminution of length with an increase of height of the wave, so that the change of length and height attend and indicate changes of depth.

In a rectangular channel whose depth gradually slopes until it becomes nothing, like the beach of a sea, these phenomena are very distinctly visible; the wave is first retarded by the diminution of depth, shortens and increases in height, and finally breaks when its height approaches to equality with the depth of the water. The limit of height of a wave of the first order is therefore a height above the bottom of the channel equal to double the depth of the water in repose. If we reckon the velocity of transmission as that due to half the total depth, and the velocity of translation as that due to the height of the wave, it is manifest that when the height is equal to the depth these two are equal, but that if the height were greater than this, the velocity of individual particles at the crest of the wave would exceed the velocity of the wave form; here accordingly the wave ceases, the particles in the ridge of the wave pass forward out of the wave, fall over, and the wave becomes a surge or broken foam, a disintegrated heap of water particles, having lost all continuity.

In like manner does the gradual narrowing of the channel affect the form and velocity of the wave, but its effects are by no means so striking as where the depth is diminished. The narrowing of the channel increases the height of the wave, and the effect of this is most apparent when the height is considerable in proportion to the depth; the velocity of the wave increases in proportion as the increase of height of the wave increases the total depth; but with this increase of depth, the length of the wave also increases rapidly, and it does not break so early as in the case of the shallowing of the water. Its phenomena are only visibly affected to the extent in which a change of depth is produced in the channel, by the volume of water added to the channel taking the velocity and form peculiar to that increased depth.

Axis of Maximum Displacement of the Wave of the First Order.

That a wave of the first order, on entering a large sheet of water, does not diffuse itself equally in all directions around the place of disturbance (as do the waves of the second order produced by a stone dropped in a placid lake), but that there is in one direction *an axis* along which it maintains the greatest height, has the widest range of translation, and travels with greatest velocity, viz., in the direction of the original propagation as it emerged from the generating reservoir, is a phenomenon which I have further confirmed by a number of experiments. This phenomenon is of importance, especially if we take the wave of the first order, the same (as I think I have established) as type of the tide wave of the sea and of the sound wave of the atmosphere. I determined this in the simplest way. I filled a reservoir which has a smooth flat bottom and perpendicular sides some 20 feet square, to a depth of 4 inches with water. In a small generating reservoir only a foot wide, I generated a wave of the first order. A circle was drawn on the bottom of the large basin, and of course visible through the water, having its centre at the place of disturbance, and divided into arcs of 30°, 45°, 60° and 90°, on which observers were placed, and the heights of the same wave, as observed at the points, is given in Table XVII. (Appendix, p. 275.)

PART II.

THE WAVE OF TRANSLATION AND THE WORK IT DOES AS THE CARRIER WAVE OF SOUND.

It was in 1834 that I first saw the wave of translation, which I also call the solitary wave. It was a year later before I had reasoned how to create such a wave, and to exhibit the phenomena I wished by the combined efforts of three horses on a channel 36 feet wide and 6 feet deep. In this channel I was able to create a large solitary wave, which travelled onward with a uniform velocity of 12 feet a second or 8 miles an hour—which continued this uniform speed over a distance of many miles with unchanged form, slowly diminishing height, and slightly diminishing speed. This wave had a peculiar shape of the cycloidal order, but radically different from any known wave. It rose above the level of the still water without the slightest hollow either in front of or behind its central height, and moved the whole mass of water embraced within its length in this forward direction, never recoiling or reacting backwards. Entering on still water, it left the water behind it equally still the moment it was passed, and did not by its mechanism of propagation part with any quantity of its moving power except the minute expenditure of atomic friction. It would therefore have in a long smooth channel the quality of transporting a large mass of water or a large measure of force to an unlimited distance with infinitesimal waste. That great solitary wave I reintroduce, because I believe that an intimate acquaintance with its nature and laws will in the future render great service to researches in physical and in chemical science. It has explained to me a multitude of phenomena, which our existing knowledge and our received laws and present theories had failed to explain, and I believe it will render great assistance to all students of the physical laws of the universe if the mode of its application to each science were understood and applied.

It is because I have in my personal department of science found the knowledge of the nature of this wave so important, both for the sound understanding and successful execution of the work I had to do, that I believe it will be equally successful in the work of others if carefully studied and thoroughly understood.

The first branch of physical science after hydrodynamics in which I believe that a knowledge of the phenomena of the solitary wave may be of theoretical value, and lead to important practical use, is the science of sound, as it is my conviction that this wave is the sole conveyer of sound to a distance, and I should like to distinguish it in the atmosphere as *the carrier wave of sound.*

It is necessary at the outset to draw a marked distinction between certain phrases in common use in this science and the carrier wave of sound, as I propose to define it.

The phrases, "waves of sound," "oscillations of air," "atmospheric vibrations," and similar phrases, are too generally and vaguely used and applied in a misleading manner. The phrases or symbols employed in most treatises to explain the nature of sound and sound waves are those of oscillation—vibration—or repetition of given motion in opposite directions. The swing of the pendulum backwards and forwards is taken as the type of these oscillations, and the following illustration is generally given as the type of the nature of the genesis of sound and its propagation through the air.

Take a basin of water or a smooth lake, drop a stone into it, and observe what follows. The stone in passing down makes a hollow in the water. This water is then filled up by water from below and from all round rushing in; rushing in it forms a heap at the centre, this heap falls down again with a swing which carries it below the level, comes up again to the surface by another swing, goes down by a third swing, and goes on oscillating up and down, above and below the level, until at last by the friction of the particles the water comes to rest.

Continuing this illustration, it is next observed that the oscillations caused by the stone on the surface spread in a circle all round, having the hollow left by the stone in the centre, and that a series of concentric rings are formed of waves in alternate heights and hollows, diminishing in height as they extend, until at last they disappear.

This is a perfect illustration of the genesis and diffusion of a series of oscillating waves as they follow in succession along the surface of smooth water moved by a fallen stone; but as an illustration of the genesis or propagation of a sound wave through the air, it is quite misleading and erroneous, and has probably done much to impede the science of acoustics. These oscillating waves are in no respect the types of the sound wave, and if the sound wave were to follow any such example, the consequence would be disastrous. In waves of this kind short ones go slow and long ones quick, and if this were true of sound waves an ear listening to music a good way off would find that all the low notes reached it first, and the high notes did not arrive till much later, and that according as the hearer was nearer to or further from the musician, so the high notes and the low notes would be nearer together or further apart, so that each auditor would hear totally different music varying with the distance.

But there is a second incongruity growing out of this illustration. One sound can reach my ear, like the tick of my watch or the report of a gun; but if the oscillating theory of propagation were true there might be ten ticks of the watch sent to my ear, or twenty reports of the gun, and we know that no such phenomena take place. Each sound is carried once to the ear and no more, and all the sounds of a melody reach the ear of a hearer each in its own place, in its due time, and all with the same speed.

I venture, therefore, to suggest that, for the future, no such illustration of the manner in which sound is carried through the air shall be given, but that the sound wave shall be treated as a solitary wave propagated in one single direction with a determined foreknown velocity, which is the same in every kind of sound, with only this distinction, that a very loud sound travelling to a great distance travels sensibly faster than a soft, gentle sound.

According to my view, each separate sound has one solitary wave of its own, which goes out from the source of sound along one straight line in a given direction, carries the impulse it receives, from whatever source, with a given velocity, and delivers into the ear the single impulse it receives. *It is thus one single complete phenomenon.*

I shall now proceed to show how a given sound can be first created, and then carried to a distance by a single wave in air of the same nature as a wave of translation in water; and I shall afterwards show how such a wave can, if necessary, be so separated into parts as to be spread abroad in many directions at once.

For the right understanding of the phenomena of sound it is necessary to distinguish between the phenomena of creating sound at its source—of conveying it to a distance—and of hearing it in the ear. These three phenomena require careful distinction.

Let us begin with two simple sounds—a clap of the hands and the stroke of a hammer; both are sounds instantaneous, without repetition or continuation. The stroke of a hammer may reach my ear in various ways. Suppose it to be given to the end of a long stick, the other end of which touches my head, the sound will be given to me from the iron of the hammer through the wood and the bones of my head as a single stroke, but if given to the piece of wood held at a distance, the sound will reach my ear through the air as a single sound just as before, only coming slower through the air than through the wood. In each case I shall hear a single stroke without repetition.

I will next suspend an elastic ball by a thread from the ceiling. The thread shall be 40 inches long. I will place a stretched membrane in a vertical plane close to this ball, and I will draw the ball aside and let it swing so as to strike the centre of the membrane. It will recoil, then fall a second time and strike a second stroke, recoil again and go on striking one stroke every second of time until the resistance of the air and other hindrances bring it to rest. There will thus be a succession of strokes and a repetition of phenomena, but each phenomenon will be quite independent of its successor, and each stroke will have sent into the air a single separate sound. It is to be observed that the mere fact that in certain cases certain causes give out a series of consequences following each other in close succession and in close resemblance, does not modify or interfere with the separate nature of each individual phenomenon. If I had chosen to stop the ball at the end of its first full swing, one perfect sound would have been delivered. This example truly illustrates what often happens in musical instruments. The successive beats were exactly timed by the length I gave to the pendulum to the number of one beat in each second of time. By making the string 10 inches long, the ball would have delivered two strokes a second instead of one, and by various other expedients any number of beats I chose could be given out with perfect regularity, and each beat would send out its own independent carrier wave into the air, and deliver each separate report at perfectly equal intervals to distances far or near.

In this example the independent nature of the creation of the phenomenon and its propagation to a distance are clearly shown. The contriver of the apparatus has the power either to create one single wave or to cause a succession of similar waves to be sent out at equal intervals, and in such manner that no single wave shall interfere with or interrupt its successor. Now there are many contrivances like this for the creation of sound, and of these one of the most instructive is the organ-pipe.

Take a large organ-pipe 1 foot square and 32 feet long. Cover the end of it with a thin membrane. Let the ball strike this drum one stroke. That stroke will require the $\frac{1}{32}$ part of a second to travel to the ear of a listener at the end of the pipe, and if the ball be now so adjusted as to strike thirty-two strokes on the membrane in one second, these strokes will give to the ear of the listener the sound musicians call C. Here, again, there is no vibration. Thirty-two successive strokes create thirty-two independent waves, each of which carries a separate stroke into the ear, which, when taken in together, make the sound belonging to the number thirty-two.

There is another way to make the same sound. Take a flat board, clap it on the end of the square pipe: the clap will push a foot of air forwards into the mouth of the pipe; that air will cause a wave to run through the pipe which will take $\frac{1}{32}$ of a second to travel, at the end of which time the wave will escape freely into the surrounding air. If at the moment of this escape I gave a second beat I should make a second wave, and on its escape a third, and so on for thirty-two waves, and if I could do all these in one second of time I should have made the same sound C which I had previously done by the beats of the ball.

But as the repetition of beats and claps in exactly measured time by human hands is scarcely possible, mechanism had to be invented to create these single air waves with exact precision of number and time. This mechanism is simple and ingenious. It is found that an air wave sent into a channel 32 feet long blocks it up, so that a second wave cannot enter till the first has escaped; a reservoir is therefore provided from which can be sent successive supplies of air into one end of the channel; we open the communication between the reservoir and the channel, and a sudden rush of air creates a wave which blocks up the channel or pipe during the $\frac{1}{32}$ of a second, and no air can enter till this block is removed; but the moment this is done the air which was pressing forward rushes in, and in its turn escapes at the far end, and the channel is now clear for a third rush, a third wave, a third block, and a third escape, and so on. The merit of this simple process is that the 32 feet length of the channel is itself made the instrument and measure for the admission of each single wave of air, inasmuch as the new one cannot begin till the old one has escaped at the end of the 32 feet pipe. The nature of this self-acting process is rendered still more instructive by a device of the inventor, which enables a pipe of 16 feet of length to do the same duty as that of 32 feet. It might naturally be expected that the wave would travel 16 feet in half the time in which it travels 32 feet, and then there would be sixty-four waves in a second instead of thirty-two, and the sound delivered would not be the same, but an octave higher. The ingenious device which prevents this is the closing of the farther end of the pipe by a flat board, so that the wave instead of escaping shall be turned back, thus travelling twice over the 16 feet, and an opening is provided through which it shall escape on its return, the going and returning occupying $\frac{1}{32}$ of a second, just as it did before.

This mode of making lengths of pipes give their own measure to times and numbers, is the principle on which great organ-pipes are constructed, and it is useful to consider wave genesis and sound genesis on this large scale, because we can examine the true mechanism of sound much better on this scale than on a minute scale.

An organ-pipe formed of a wooden channel 64 feet long and 4 feet square is a most instructive instrument. The air makes only sixteen beats a second, sending sixteen waves a second, and each wave travels at the rate of 1024 feet a second, whether the pipe be large or small. Thirty-two waves along a 32 feet pipe in a second, and sixteen waves along a 64 feet pipe in a second, make up the same journey of 1024 feet in a second.

Waves.	Distance.	Time.	Speed.
32	32 feet.	$\frac{1}{32}$	$32 \times 32 = 1024$
16	64 „	$\frac{1}{16}$	$16 \times 64 = 1024$

The reservoir of air for the genesis of waves sent into channels of given length, and therefore measured in time by the successive blocks in each channel of one wave at a time, is kept full by the organ-bellows, and the pipe gives the time of the blocks. But cther methods may be chosen to cause exactly the same number of waves, and measure them out in the same equal time, and by these methods the same sounds can be produced without the intervention of the organ-pipe.

To do this a reservoir of air is provided, a flat board with a hinge is also provided, also a machine to open and shut that door thirty-two times in a second. Each time that door is opened a rush of air takes place and creates a wave, which is sent out from the open door, and travels with the standard speed of 1024 feet per second. Between the wave thus created by the mechanism of the opening door and the measuring lengths of the organ-pipe there is no practical difference, and the organ sound is thus produced without the organ.

The same result has been obtained by another method, and without the merit of invention. It has existed from time immemorial as the music of the reed, and is produced in modern instruments by a simple steel spring. This elastic flat piece of steel is used to cover an opening; it is fastened to one of the four edges of that opening, so that by its elastic force it can open and close the opening. What happens is this. The air from a reservoir passes through the steel door, either to open or to shut it. The maker of this arrangement has to proportion the thickness or thinness of the steel plate in such a manner that the recoil of the spring, when closed or opened by the force of the air, shall be of that exact force to close and open it thirty-two times in a second. It thus sends out thirty-two pulses of air, which create thirty-two successive waves, which, travelling at the rate of 1024 feet a second, give forth the note C just as given by the organ-pipe.

I have thus produced all the sounds of renowned musical instruments without once having recourse to the oscillations of positive and negative waves, which are erroneously supposed to constitute the essence of sound and sound waves, and which are repeatedly alleged to be formed on the model of undulations on the water from the falling stone.

But I must now enter upon the more practical question of the method of propagation of such sounds as have their origin in unquestionable cases of oscillation, and in those cases also I shall show that although the causes of sound genesis may be found in the positive and negative oscillations of certain bodies, yet that the nature of the cause and the nature of the effect *may be, must be*, and are totally different.

The vibration of a stretched string and that of a tuning-fork are familiar examples of oscillatory movements back and forward, up and down, just like the waves of water repeating each other in alternate heights and hollows. These oscillations to and fro of the stretched string or the steel fork do unquestionably move the air in alternate oscillations, repeating themselves like the oscillations on a pond disturbed by a falling stone. To this I offer no objection. What I allege is, that the motion just described is not the motion which gives to the ear the sensation of sound. The movements described are movements merely local; they remain near the string or the fork; they stir the air round about them just as the stone stirs the water round about it, but this local movement is quite of another nature than the motion which must be generated in order to carry the effect of a sounding body away from itself and deliver it to the ear of a listener at a distance; and I will show that in order to do that means must be taken first to create a series of solitary waves, and then to make these waves act as carrier waves, transporting the sounds from the instrument to the ear at a distance. In other words, the operation carried on in the instrument differs from the operation carried on through the air and from the effect delivered by the carrier wave.

The question I have to solve is this. How can a set of local agitations and vibrations to and fro be so employed and applied as to create uniform series of solitary waves carrying sound in equal time and measure to a distance?

In the first place, it is easy to prove that the small oscillations do not and will not form sound waves, nor will they travel to a distance. A well-known experiment proves this. The tuning-fork, when struck, vibrates strongly, but does not send forth its sound to a distance; a music string, stretched by a heavy weight, held up by the hand, and plucked like a harp string, oscillates and vibrates, but the air refuses to carry its oscillations and vibrations to a distance. Some new means must be supplied before the air can be persuaded to take up out of the string its moving force and carry off audible sounds from it to the distant ear. The carrier wave must first be created.

We have now reached the question, how can the carrier wave be created by the vibration of a string which of itself cannot create such a wave? To find that out let us go back to the organ-pipe. The first way in which I sent out a wave from the organ-pipe was by taking a flat board, and giving it a successive series of pushes forward, so as to send forward at each push a solitary aerial wave; and in another such case I took a stretched sheet of parchment, gave it a succession of equal timed strokes with a ball, and thus sent out wave after wave of air to a distance. I must now invent a mode by which a vibrating string or an oscillating tuning-fork can send forth a succession of equidistant equal-timed waves.

For this purpose I must have recourse to my stretched membrane, to a flat movable board, or to some artifice of that nature; and I must so arrange it that each vibration of the string shall send a throb of force into the board which shall push it forward against the air, thus sending an aerial wave forward through the air exactly as in the organ-pipe. Now this is what is most ingeniously done in stringed instruments. The strings of a guitar might be stretched on an open frame, dexterously struck into perfect vibration and well timed by the skill of a musician, and yet totally fail to send a single wave of sound to a listener at the end of the room. The sounds have been created, but the carrier wave has not been created. If a sheet of parchment be stretched on a hoop large enough to have vibrating strings stretched across it, then when one string is plucked and let go the reaction of the striking force will give to the stretched membrane a quick short impulse forward, instantly generating an aerial wave going at right angles to the stretched membrane at the rate of 1024 feet a second, and each succeeding vibration will send out each successive wave in like manner. This is the primitive instrument of the East. The guitar is another expedient of the same kind, but more refined and complex, and the vibrations of each string move a flat, thin, elastic wooden board, which is made fast upon a hoop or round frame kept in shape by a strong wooden back. This sounding board sends out by each pulse it receives from a string an aerial wave which carries its music to a distance. A violin, a violincello, and a piano all mainly depend for their excellence on the quality of this board, therefore an ancient violin with a good board is a matter of great price, far more costly and difficult to obtain than the most perfectly vibrating strings.

ON THE ANALOGY BETWEEN THE SOLITARY WAVE IN WATER AND THE SOUND WAVE IN AIR.

THE genesis of sound, its transmission and its sensation, are three phases of the phenomena of hearing which (in order to be investigated and clearly understood) should be as completely separated in the mind as though they were three independent phenomena. What happens at the source which gives out the sound, what happens in the ear which receives it, and what happens in the process of carrying it, form three independent subjects of investigation.

Let us examine these phenomena as they take place in water. The solitary water wave is the only hydraulic phenomenon which performs the function of receiving certain mechanical power at one place, and delivering it unchanged at a distance, so that work shall be done by the original cause at a remote distance if required, in the same manner as it would have been done at the origin.

The genesis of this wave is as follows:—A sheet of still water extends a length of twenty miles. Call one end of this lake the starting station and the other end the delivery station. I possess a large reservoir of water, shut off above the level of this lake. This water I could employ to drive a wheel or to pump water to a high level, but I wish this work to be done twenty miles off. I have 1000 tons of water, and I wish to make the 1000 ton-power work for me at twenty miles' distance. I have only got to deliver the whole mass into the lake at one moment. As the lake is 32 feet deep, in one hour from the water quitting the starting point, 1000 tons of water will be seen arriving at the other station, and if there be arrangements made to receive them they will at once do the work intended.

It is important to notice that in this transmission there has been no loss or waste of power, with the slight exception of a small waste by friction. The phenomenon as seen would be as follows:—The 1000 tons of water delivered into the lake would form a heap 100 feet forward from the sluice. This heap would be 2 feet high along the ridge, the length of which is 360 feet, sloping forwards and backwards over a breadth of 200 feet. In the next instant this heap is seen to be rapidly changing its place but not its shape: it moves uniformly at the rate of 32 feet a second, and is found at the end of an hour to have reached its destination at this uniform rate of 32 feet in a second.

A reservoir has been prepared to receive it, into which it leaps and spends itself at once on the work ready for it, or it can remain stored up to do work at leisure.

An important question here meets us. Might not the same useful work have been done by allowing the 1000 tons to flow slowly and gently into the lake in a continuous stream, and thus by raising slightly the level of the lake have accomplished the end in view? The answer is, that the diffusion of the 1000 tons store would have wasted nearly the whole of the useful power in the water, and scarcely an appreciable quantity of power would have been received at its final destination.

We will next examine how this motion of transmission of so great a force by so great a mass of water took place with such high speed and so small waste. First, none of the water sent into the lake at one end was sent forward to the other end. The whole lies at perfect rest where it fell, and all that it did in falling was to raise the mass in front of it up into a wave consisting of exactly the same quantity as itself, atom for atom. This second mass of displaced water (forming the wave heap in advance) travels only so far forward as the pressure behind compelled it, and then subsiding gave out its motion to the water in front of it, compelling that in turn to form a third heap, and from the ridge of that third heap the water again falls to rest, giving out its motion to a new mass, and this process goes forward all along the lake.

We have next to seek the cause of the particular speed. of 32 feet a second. It is found, strangely enough, neither in the quantity of water nor in the speed of the motion with which it entered the lake, nor in any peculiarity in the source of motion, but alone *in the depth of the water*, and that is one of the most important points in the relation of the wave in water to the wave of sound, as it will be found that the depth of the aerial ocean bears as close a relation to the speed of propagation of sound as the depth of the lake bears to the transmission of power by the solitary water wave. We must examine into what happens in the lake when under the influence of a wave, and how its speed increases or diminishes with the varying depth of the water.

It might be naturally thought that it would be harder to drive the larger mass of deep water forward than the smaller mass of the shallower lake, and that, therefore, the speed would be greater in the shallower and less in the deeper water. The contrary is the fact. A lake four times as deep sends its waves forward twice as fast, and nine times the depth sends the wave forward thrice as fast, and a sea one hundred times as deep would send it ten times as fast, and in that case the twenty miles we have done in an hour would have become two hundred miles, and that is only about one-third part of the speed of sound.

We must look for the cause of this in the nature of the motion which takes place in the inside of the wave. Looked at from above the surface, the wave seems to be made of water rushing forward with high speed; but looked at from down below, the real motions of the atoms of water are found to be quite different from the seeming motions of the wave. Seen from below each atom is observed to be first lifted gently straight up, then moved gradually slowly forward, growing quicker and quicker until right under the crest of the wave it moves fastest forward, then slower and slower, and at the end moving slowly down to its old level, where it remains at rest. Thus four different processes take place during the transit of the wave. First, starting up and forward; second, gradually quickening to the highest speed; third, gradually slackening speed; fourth, stopping. Simultaneously with these motions forward are taking place different motions upward. First, gently rising; second, springing up and stopping; third, slowly falling down; fourth, coming to rest on the original level. These two sets of motion constitute the process of wave genesis as it can be seen by the eye which watches each water particle during the wave transit; and the complete understanding of this process is necessary in order to comprehend the constitution of the wave.

All these changes are simultaneously going on in different degrees throughout the mass of the fluid, whose elevation and wave form are seen above the surface, and all the columns of particles in it—in one line from right to left, and vertically up and down—start forward at the same instant, march in line, and stop at the same instant. But, though all alike in right to left and forward motion, they differ in up and down motion. Those near the bottom rise up and descend through a small space; all (say 1 foot from the bottom) rising and falling 1 inch, and all 12 feet from the bottom rising and falling 12 inches, while those on the surface rise and fall to the exact height of the wave itself.

This measured methodical and somewhat complicated motion may be said to form the mechanism of the wave, and all the particles which constitute it may be considered as the individuals of an army performing methodical manœuvres in systematic line and order. These motions in the water wave are an exact type of the motions in the air wave.

In the methodical nature of these manœuvres we will now search for the cause why the same force employed in wave genesis should result in a rapid moving wave in deep water, and a slow moving wave in shallow water; or why it should communicate a motion twice as quick to a mass of water four times as great. We can only understand this by following out the perfect contrast which exists between the two natures of motion concerned in wave creation and wave propagation.

We have seen that when the wave travels across a sea from one side to another, the water making that wave does not travel, but stays behind, and the contrast now to be added is this—that in the quick moving waves seen on the surface the under water particles move slow, while in the slow moving wave in shallow water these under water particles move quick. Thus the wave motion and the water motion may be of quite opposite natures.

We must see how to reconcile these opposites. When I throw a large mass of water into one side of a deep reservoir, and thus lay on the top a heavy weight, that weight quickly makes itself felt as a burden on all the columns of water standing directly below it; to this strain the vertical columns gradually yield, and as they cannot go into the side of the reservoir, they go forward, forming a vertical wall of water, extending from top to bottom of the reservoir. Now the mass of added water will in diminishing send this wall forward through a smaller space in proportion as the water is deeper, because a large area moved a little way forward will make room for the added mass of water, while a smaller area of the shallower water would require to move much further forward to make room for the same added mass. Therefore the motion of the *particles* in the wave is shorter, slower, and smaller in deep water than in shallow.

But a deep water wave has another quality which a shallow water wave has not. The deep water wave moves much quicker than the shallow water wave—first, because the distance to go is less, and second, because the force to move it is greater. The force of a column of water is 64 lbs. on the foot for every foot of depth, and therefore at 10 feet deep the force of a column of water pushing another forward would be 640 lbs. of pushing force; at 20 feet deep it would be 1280 lbs., and at 40 feet it would be the force of a ton on the foot. This twofold proportion of lesser weight to be removed and of greater force for its removal gives increased speed in a twofold proportion, which we may call the duplicate or square of the depth.

Equivalent Oceans.

It may help here to elucidate the subject we are treating that we should conceive three equivalent oceans, and investigate their phenomena. A sea of water 32 feet deep is the equivalent of a mass of mercury or of molten metal 28 inches deep, and of an ocean of air 5 miles deep. These three seas or oceans being of equal weight, contain the same quantity of matter in a given vertical column. It therefore follows as an elementary truth that any given force applied to produce motion in one column of the one fluid would communicate to it the same motion as to an equivalent column in the other fluid, and careful experiment has shown that like mechanical force applied in like manner to all three seas sends forward through each a single solitary wave moving with a uniform speed, and delivering the force communicated to it into whatever receptacle has been prepared for it.

But between the three oceans would be this marked distinction, that though the mass to be moved, the force used to move it, and the resulting delivery at a distance may be alike, the times of delivery would be different. The wave which would travel in the water ocean 32 feet per second would travel through the aerial ocean 880 feet in a second, and through a mercurial ocean 12 inches a second. Thus the phenomena of equivalent ocean weights seem in the matter of wave transmission to be quite discordant.

We will proceed to their reconciliation. We must find an ocean of water as deep as an ocean of air (say 24,000 feet deep), and we must imagine an ocean of mercury of the same depth. The remarkable reconciliation will be this, that in these three oceans a given wave will travel with exactly the same speed. We have no means of testing the accuracy of this assertion in a mercurial ocean, but we have the means of testing it in the water ocean, because the sun in his daily revolution round the earth makes in passing over the ocean one great solitary wave of translation, and the moon in passing over the ocean also creates a second solitary wave of translation, and these two waves are transmitted from shore to shore with exactly the same speed in water with which the sound wave is sent through the equivalent ocean of air. I am so impressed with the truth of this law, that the velocity of this solitary wave in any fluid is due to the depth of the fluid in which it moves, whether thick or rarefied, that I hazard the hypothesis, that in the unknown element which pervades the universe, and which, though unknown, is the cause and medium of the most familiar phenomena of everyday life, proceeding on the same basis of calculation as in the water and air occurs, we shall find that the ethereal ocean should be given a height of 5,000,000,000 miles, and that the corresponding velocity of the solitary wave through that ocean would be 1,000,000,000 feet per second, or 1,000,000 times faster than the speed of the corresponding wave in air.

If we consider these four oceans of metal, water, air, and ether as elements for the propagation of waves, each ocean of unlike nature but all governed by like laws, we shall be able to understand how the phenomena of earthquakes, tides, sound, light, heat and electricity are all the result of physical forces stored up, then suddenly set free and carried by a wave of translation through its own ocean to the place of its destined effect. Thus everywhere in one shape or another these waves are doing the work of the universe, each by different means, but all by the same law.

Equivalent Oceans.

Water ocean	33 feet deep.
Mercurial ocean . . .	29,1258 inches deep.
Air ocean	26,400 feet deep.
	Five miles high.

So long as we consider only one wave transmitted through an ocean of unchanging depth and like material we can calculate with certainty the speed of the wave, but if we conceive the wave generated in one fluid to pass into the ocean of another fluid we must foresee that change would be inevitable. The atoms of the lighter fluid would be incapable of giving equivalent speed to the atoms of the heavier fluid, while the atoms of the heavier fluid might give much more than their own speed to the atoms of a lighter fluid. Also in the transmission from one fluid to another, part of the motion of the one might be given off to the other and part retained. Likewise if one portion of a fluid were in a different condition from another portion of the same fluid, by heat, motion, or some other cause, the transmission might be hindered, accelerated, changed in direction, created or stopped, and from these differences a great variety of wave modifications would follow.

Another cause of wave modification is this. A channel or reservoir of water may have a given depth and nevertheless have a varying form. A lake may be deep in the centre and shallow towards the sides, or the contrary, and thus an unseen cause would become an element in the nature of the wave so as to double or halve the velocity of its speed.

These elements of wave variation in the deep ocean become equally serious and of a different nature when we pass from a deep water ocean to a deep air ocean. In the atmosphere we study its phenomena from the bottom while its variations are going on from the top, and we can only infer what the phenomena probably are which are going on at the top. What we do know is, that a given quantity of air down below occupies a larger and larger place as it rises up. We know that a thousand gallons of air if taken to one-eighth of the calculated height would increase in bulk to 1150, and carried half way up would become 2000 gallons, and at heights which we have not reached would swell out into 4000 and 8000 gallons. Thus the same weight of air and the same number of atoms taking quite different bulks may also take quite different states, and so in the remote heights of the atmosphere the same enormous variety of conditions might exist as those caused in the bottom of the water ocean by the configuration of the land on which it rests.

But there is one cause of approximate uniformity in the aerial ocean which mitigates such difficulties in the higher regions. While a given volume of air may change enormously its bulk, as it lies lower down or higher up in the atmosphere, that same mass cannot change its weight, and as we are always able at the bottom to measure the whole weight above us, we can by that means measure accurately the forces which propagate the air waves along the bottom of the air ocean in which we move. These calculations we must now proceed to make.

The great propagating force of the aerial atmosphere being its elastic or pushing power, and that pushing power being always accurately measured by the weight resting above it, we are led to the conclusion that the pushing power of the atmospheric ocean must diminish in exact proportion as the weight pressing on it diminishes: therefore that portion of the atmosphere which is high up can have very little effect on the propagation of a wave low down; in other words, the elastic force diminishes in the exact ratio of the superincumbent weight. Now, in the water ocean this is quite different, for in it the weight does diminish upward in successive layers of equal weight, but the pushing force remains sensibly the same from top to bottom, being due to other causes, while the pushing force of the air is due to weight alone.

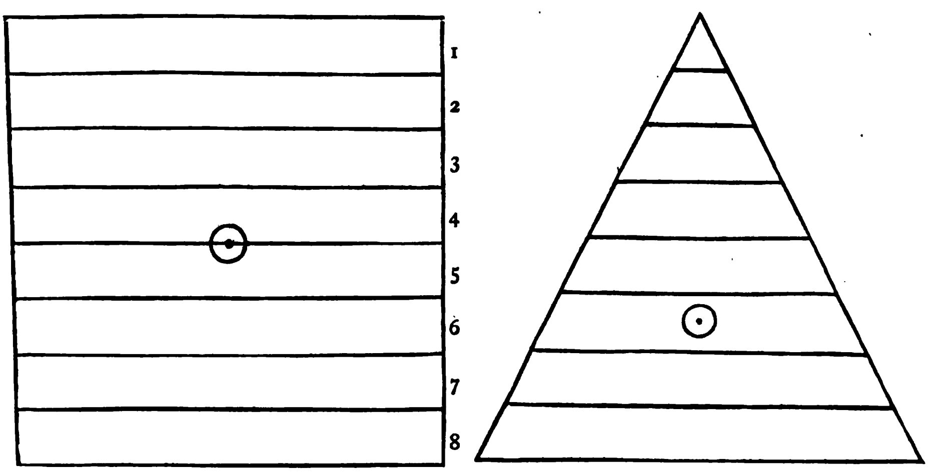
The consequence of this difference is, that we must measure the effect of the propagating force of an aerial wave by a power diminishing as the height increases, and this diminution will be uniformly measured by the diminished weight of that height.

Let us now take the ocean of water and its equivalent ocean of air, and compare the effects of what we have been considering in the two. Take a convenient height of 32 feet as the water ocean, and the corresponding height of 25,600 feet as the air ocean. Conceive the water ocean divided into eight horizontal strata, each 4 feet deep; the uppermost 4 feet below the surface, the second 8 feet, the next 12 feet, and so on through 16, 20, 24, 28, 32. As each one of these occupies the same bulk, has the same elastic force, and the same power to move and be moved, the propagation of the wave increases in a given ratio to the depth without variation caused by varying condition.

On the other hand, let us divide the air ocean with the same number of level strata of equal weight, each stratum weighing exactly the same as the 4 feet stratum of water. Between the weights of these successive strata of air there would be no difference from the equivalent strata of water; but in pushing force or elasticity there would be a radical difference, for the pushing force of each successive stratum of given weight of air would become less and less in exact proportion to its order of stratum, the pushing force of the lowest stratum being eight times that of the highest, diminishing upwards in the proportion of 8, 7, 6, 5, 4, 3, 2, 1.

The consequence of this graduated diminution of force in successive upward strata is, that we must seek the measure of effective force in wave speed lower down in the air ocean than in the equivalent water ocean, this proportion being that in the water ocean we took one half of the depth as our measure for speed, whilst in the air ocean we take two-thirds as the equivalent depth.

Geometrically this is represented in the following manner:—



These two diagrams represent by their successive depths strata of equivalent weights in water and in air. The pushing power is represented by the horizontal lines, which remain unvarying in the water ocean and diminish gradually in the air ocean, the mathematical consequence of which is that the centre of effect in the one is half-way down and in the other two-thirds. The water wave will therefore be measured in speed by the velocity acquired in falling through half the depth, and the air wave by the velocity acquired in falling through two-thirds of the depth.

The condition of our atmosphere being affected by the weight of the upper air and by the effect which that weight has in regulating the elastic force by which the sound wave is propagated, is the reason why we have dealt with the atmosphere as though it were an ocean of uniform depth, and of weight increasing exactly with that depth. Hence we have obtained a velocity for the air wave which is precisely that which experiments in sound have given. These results are as follows :—

32 feet of water ocean.
32 × 800 feet of air ocean = 25,600 feet high.
Speed due to two-thirds of 25,600 feet high = 1045 feet.
33 feet water gives . . . . = 1060 feet.
34 feet water gives . . . . = 1077 feet.

Sound by experiment varies with the barometer, or mercury and water measure from 1024 to 1096 feet speed.

We now know that our atmosphere will carry an impulse given to it in the same manner as a sea of water or the ocean will carry an impulse given to it, and whether that impulse be violent or gentle, will carry it with a given speed, and that speed will be proportioned to the nature or condition of the atmosphere or the air ocean. We will now consider the circumstances in the condition of the atmosphere which produce variations in the normal speed of this impulse, which variations, however, are never such as to make the sound travel less than 1000 feet a second, or more than 1100 feet. The conditions which may cause a variation of 10 per cent. from the highest to the lowest speed of sound are those indicated by the barometer, which marks the changing weight of the atmosphere. The passage of the sun and moon over our heads and under our feet cause ebbing and flowing in the ocean of air just as they cause tides in the water ocean, and as a solitary wave in water or air travels faster in greater depth and slower in shallower depth, so a sound wave carries sound faster when the barometer shows that the ocean of air is deeper, and slower when the barometer shows it to be shallower.

Another cause changes the condition of the atmosphere. The same weight of air will form a deeper ocean when hot than when cold, but it does not always follow that in cold air sound will always travel slower than in hot air, because in our atmosphere the condensation of cold air may cause a rush of air into the cold place, and a rush of air away from the hot place, and so cause a high barometer where the thermometer was low and a low barometer where the thermometer was high. These opposite conditions may therefore agree with, assist, oppose, alter, or counteract each other, and although heat may cause the variations the barometer will be the safer measure of the condition of the atmosphere for the transmission of the sound wave, which will always travel faster with a high barometer and slower with a low.

There is still another cause of slight variation in the velocity of sound, namely, the nature and measure of the force which creates it. As a general rule, all sounds of the voice and of musical instruments are created by forces so gentle in comparison to the mass of the atmosphere, that their effects do not alter sensibly its condition, but a violent force like a clap of thunder or the explosion of a large magazine of powder does for the moment so modify the condition of the atmosphere as sensibly to affect the barometer. If we use a water barometer 32 feet high, the report of a cannon would make it vary an inch or two, and a barometer made of enclosed air would show a sensible compression. This increase of velocity has thus been made appreciable in sounds of extraordinary loudness only.

Although, as I have said, we live at the bottom of an ocean of what we call air, that ocean is made up of several substances. There is one ocean of oxygen, a second of nitrogen, four times as large, and there is a sea of vapour or water air of continually varying quantity. Possibly, also, there may be a good deal of pure hydrogen and some other gases, but the three great atmospheres pervading our aerial ocean are oxygen, nitrogen, and vapour. These three atmospheres all consist of atoms of matter of quite different natures, and as Dalton and other philosophers have shown, each of them pervades space separately and independently. Each atom of these elements has a clear vacant space around it of more than one thousand times its own bulk, and all the atoms of one kind, standing off from the others at this great distance, leave plenty of room vacant and free for the occupation of any other groups of atoms which may have force to occupy that space, and which may have like but different distances between their own atoms. We may readily conceive three entirely different atmospheres pervading the space which we call our atmosphere—being everywhere—leaving each other quite free, and yet in some mode affecting each other.

This triple atmosphere has much to do with the propagation of sound.

Let us consider them in their independent condition. If there existed no atmosphere around us but that of oxygen, our mercurial barometer would only stand 6 inches high instead of 30 inches, and would only rise one mile high in a uniform ocean, and in the rarefied state would be only one-fifth of its present density. Sound in this oxygen atmosphere would only travel at the low speed due to one mile of height.

The second independent atmosphere, that of nitrogen, if it existed alone would show a barometric height of 24 inches of mercury, but it would rise to four miles as an ocean of uniform density, and sound would be transmitted through it at the speed due to four miles.

The third atmosphere, that of vapour, is radically different from the other two, as it exists in a state of continual variation caused by, and itself causing continual changes of temperature. While we may consider our air ocean as having a tolerably level surface above, like that of the sea, we cannot consider the vapour ocean as having any such level surface; we must, on the contrary, consider it as rising and falling in an endless series of undulations, as being also here a very dense atmosphere, and there a very rare one; and further, we must recognise in this vapour its constant tendency to change from the state of a pure vapour into water-films, air-bubbles, and rain-drops, and in all these phases it will pass out of the region of a perfect conductor of sound into that of a positive non-conductor.

Happily these three are so nicely mingled in our atmosphere that they practically form but one, and though uncombined (chemically) are physically intermixed, but leave free spaces between them and mutually act and react upon each other by attraction and repulsion. It is worth observing that ascending upwards the vapour atmosphere will be the first to disappear. The oxygen will be the second, and in the higher regions only the nitrogen will remain; a disposition evidently in harmony with the necessities of animal and vegetable life.

Table of Speed of Sound Wave in Variable Air Ocean.

Mercury.	Water.	Air.	Sound-speed.
			Ft. Sec.
26.4780	30	24,000	1011
+ 27.3606	31	24,800	1028
28.2432	32	25,600	1045
29.1258	33	26,400*	1062
30.0084	34	27,200	1077
30.8910	35	28,000	1093
+ 31.7736	36	28,800	1108
32.6562	37	29,600	1124
33.5398	38	30,400	1139
34.4224	39	31,200	1154
...	...	31,680†	1162
35.3050	40	32,000	1168

* Air ocean five miles high. † Air ocean six miles high.

In this table are some remarkable phenomena. An air atmosphere of five miles in depth gives a velocity of sound 1062 feet a second, and an atmosphere of six miles gives a velocity of 1162 feet a second, a difference of one mile in depth giving only a difference of speed of 100 feet in a second.

A second point worthy of note is the change of velocity in sound which attends the ordinary changes of weather as shown in the barometer. We may consider 27 inches to 31 inches as extreme ranges of the mercurial barometer. Thirty-one feet on the water barometer is equivalent to 27.36 inches on the mercurial barometer. Thirty-six feet on the water barometer is equivalent to 31.77 inches of mercury. To these numbers correspond 24,800 feet of equivalent atmosphere and 28,800 feet, and the two corresponding sound speeds are 1028 and 1108, being a difference of 88 feet a second, so that in the extremest oscillations of weather the change of speed of sound may be 80 feet a second, or a variation under 8 per cent. Attention may here be called to another phenomenon in the nature of certain exceptional sounds which will cause an exceptional variation in speed. A sound is created by forces sufficiently powerful to change the condition of a large portion of our atmosphere; such a sound may be caused by some great explosion, as of a volcano or mine, or even of a large cannon, sufficient to raise the air ocean for an instant 1000 feet above its standard height. This report would travel 17 feet a second faster than a common sound, and at a mile distance would be 84 feet ahead of the normal distance, and at 10 miles off would be 840 feet ahead, and would thus arrive nearly one second before its due time. This is the explanation of the experiment made by Captain Parry on the ice of the Polar region, where he ordered a cannon to fire in the direction of observers at a distance, and they heard the report of the gun *first*, while the order to fire reached them *second*—in other words, the loud sound outstripped the gentler sound.

We have now spoken of the exceptional causes that affect the speed of a sound wave, and have seen that a force powerful enough to elevate our atmosphere by 1000 feet was only able to quicken the sound by 16 feet, that is by $\frac{1}{64}$ part of its speed. Some philosophers have supposed that every source of sound has a special power to change the condition of the atmosphere, so as to alter the speed sufficiently to quicken it by over 140 feet a second. The power required to do this would be that necessary to raise the height of the atmosphere from 24,000 feet to 32,000 feet. In order to accomplish this the unseen agency of heat has been called in, and the quantity of heat necessary to do this would be the quantity necessary to raise the surrounding atmosphere from freezing to boiling point. That every gentle musical sound should have sufficient strength to place the whole of the atmosphere between the source and the receiver of the sound in this condition of intense heat or of violent strain, is too unreasonable to be listened to for a moment, if it had not received the sanction of distinguished names. No such fiction is needed, for the solitary carrier wave of sound does that work with exactly the velocity which experiment has shown to be the true speed of sound, namely, above 1000 feet a second.

It may be mentioned here that even Sir Isaac Newton's calculations of the speed of sound fell 100 feet short of the truth, and therefore corresponded to an error of a mile in the height of the atmosphere, and that he could invent nothing better to account for the error than this sudden inflammation of the atmosphere. To this the reply is, that the existence of the solitary wave of translation was not known to Newton, that the nature of its genesis and propagation could therefore then not be calculated; but that present knowledge of the nature and laws of this wave completely explain and accurately measure its phenomena without the introduction of any hypothesis contradicted by fact.

The inside of the Carrier Wave and what is going on there.

Nothing can be more vague or misleading than the manner in which the words wave, wave length, wave oscillation, are being used both in the sciences of light and of sound. A wave of the ocean has, as we have seen, two different aspects, the one presented to the spectator looking at it from above and the other to him who regards it from below the surface, and it seems not improbable that the one aspect is radically different from the other. We will take an ordinary sea surface wave; what is seen is a ridge of water rising between crest and hollow 8 feet, from crest to crest 60 feet long, and travelling its own length in 4 seconds or about 10 miles an hour, and the form of curve which the spectator sees has a beautiful shape, sharp at the crest and sloping from it gently towards the bottom of the hollow. To the eye this great mass of water seems to be actually travelling at this great speed, but does not so travel. A bit of floating sea-weed shows that while it goes forward with the crest of the wave it returns back in the hollow of the wave, and in deep water it goes each time as far back in the hollow as it was taken forward in the crest. What is truly seen by the observer is this. He sees a form going steadily forward from place to place unchanged, but he knows that the water itself does not travel with this form, but only goes forward so much and backward so much and so remains near its own place. Let the same spectator descend in a diving dress to the bottom of the sea, and no wave is to him anywhere visible. When he is not far from the surface he receives a push from the water forward as the crest passes over his head, and he receives an equal pull backward as the hollow follows the crest, and as he goes downwards to the bottom the push and the pull each become gentler, and in greater depths become insensible.

The careful observer must now supply himself with some floating particles of the same weight as water, and these he must sprinkle throughout the water which surrounds him. If he is 10 feet under water he will observe each floating atom move forward when a wave crest passes over his head, and backward when the hollow passes over his head. He will next observe that the atom moves upward under the face of the wave and downward under the back of the wave. Watching these motions more closely he will find that these four motions are really four parts of one and the same circle, and that each water atom makes a complete revolution in a vertical circle, which it repeats with each successive wave.

When the observer descends to the depth of 20 feet he finds the floating atoms describing similar circles, only they are now but half the diameter; but they keep time with those above them and with the moving wave along the surface. Descending to 40 feet it is found that the circle is still more diminished in diameter, and this continues as the depth increases until the motion dies out.

These therefore we may call local waves, insomuch as they do not remove the water they agitate out of its place, and the oscillations which appear to travel are only the seeming motions of a form repeated over and over again in the same place.

We will now examine under water a quite other nature of wave, not one repeating itself over and over in one place and so seeming to the eye to travel, but a single wave form travelling alone without follower or predecessor, and carrying with it some latent power from beginning to end of its journey, however long. To the eye above the water there may be little apparent difference between the form of this solitary wave and that of one of the oscillating waves; it will have a crest as its centre, a front sloping down forwards and a back sloping down backwards, but there will be *no hollow* below the level either before or behind; the wave will pass by a gentle curve into the straight line of the still water level, and the water out of which the wave has passed returns to its state of dead level. This solitary wave may also be 60 feet long and may also travel about 10 miles an hour, but the characteristic between the two different kinds of waves is to be found under water.

The observer who goes under the surface of the water and distributes his floating marks around him, finds to his surprise that none of these revolve as before in a circle. He next observes that they all make one single step forward and stop; also that the step of each particle begins when the extreme front of the wave has been reached, and ceases the moment the extreme back of the wave has left it. He also observes that this single step is absolutely all that happens to the under water particles. His next observation is, that there is no change in the length of the step forward as he, the observer, follows the movements from the surface of the water to the bottom, there is no diminution in descending such as was seen in the oscillating wave. When this solitary wave has a small height each step forward is a small one (say a yard), and when the height is greater the step forward is larger (say a fathom), but in each wave every particle from top to bottom starts forward at the same instant, makes exactly the same length of step, and stops at exactly the same instant. Thus the two waves are radically different.

There are minor distinctions between these two orders of waves which must not be overlooked. We may see on the top of the water a solitary wave 200 feet long, and we may notice it travelling forward at the rate of 20 miles an hour, and we may select a wave of the same length out of a group of oscillating waves, and find that its seeming velocity is only 17 miles an hour. (The length of an oscillating wave is found to have a definite relation to its speed, and we calculate the speed of surface sea waves by a standard measure of 3.57 feet. Thus in a given sea all the surface waves will be seen travelling with different speeds according to their differing lengths.) But in that same sea there is only one speed for the solitary wave, which is that obtained from the depth of the sea.

There is another minor difference between these two waves. The height of a solitary wave has to be added to the depth of the water in calculating the speed of the wave, whereas in oscillating waves the speed varies only with change of wave length.

When we go back to the aerial atmosphere or air ocean, we find the same phenomena take place there as in the water ocean, only at the bottom instead of at the top. In that fact we can at once see why all sounds must be propagated in solitary waves, instead of by oscillating waves. Oscillating waves do not reach the bottom of any deep ocean. They represent mere local disturbances, and each local disturbance has a special speed of its own and a local wave length of its own; also its disturbance becomes less and less as it is nearer to or further from the centre of local agitation. But the solitary wave moves the air atoms forward through an unvarying distance, travels with an unvarying speed, and so delivers all sounds committed to it in exactly the same time, order, and speed in which they were given. It is thus the only possible instrument for the transmission of musical sound.

The one phenomenon which we at the bottom of the air ocean can observe, during the transmission of a carrier wave, is the fact of the step taken forward by each particle of air during the transmission. Each particle is started, sent forward, and stopped. This is one complete phenomenon, and its effect upon the ear is to give to each rank of particles of air in the entrance channel of the ear a like simultaneous step forward. This simultaneous march in the channel of the ear is suddenly stopped by an upright partition on which they all at once deliver their impulse; and to the mechanism for the further transmission of the carrier wave we must give careful investigation, for this mechanism is admirably suited to the propagation of sound by a wave of the first order, or, as I have called it, the carrier wave, and is in no way adapted for the reception of the oscillating or secondary class of waves; but the consideration of this must be postponed till further on.

Musical Sound.

From an examination of the nature of the atmospheric wave of translation, and the work it does as a carrier wave of sound, it might be inferred that there is a monotonous sameness and want of variety in the sounds carried by this wave, which may seem at variance with the infinite variety in quality, nature, and effects which are found in those melodic harmonies that we call music. The erroneous theory (as I call it) of musical sound is that which supposes it to consist of oscillations and vibrations formed on the model of the waves in a pond oscillating to and fro, and vibrating up and down, and spreading out all round in circles, each size of wave having a peculiar speed and space of its own.

My theory, on the contrary, is that all sounds are carried through the air alike with one same speed, in one same kind of wave; that the wave moves always one way forwards, never oscillates forwards and backwards; and that all musical sounds and all musical relations grow out of this uniformity.

I nevertheless admit the obvious fact that vibrations like those of a piano string may be employed as a mechanism by which the genesis of sound waves may be accomplished.

They may give to what is called a sounding board the impulse required to create each wave of sound, and send it forward, but the vibrations end there; and what is carried forward and passed into the ear has no resemblance to the oscillations of the string which was employed to move the board; and I need scarcely add that the phenomena in the labyrinth of the ear have as little likeness to the vibrations of the piano string employed to send out the wave as the report of a shot has to the cannon which fired it.

I must now face the question, How does the sound wave convey musical impressions, if it does not carry the oscillations of the string?

The answer I give is, that the sound wave carries precise measures of time; that just as exact dimensions of size and form create a piece of sculpture, and give an impression of beauty to the eye, so do exact measures and relations of time give the impression of beauty to the ear.

Time and time-keeping is the sole element of musical sound. I will begin with the simplest elements of music. The beat of a drum, and men marching to its time, the twang of a tambourine, and women dancing to it, are rude samples of the pleasure arising out of keeping time. The pleasures of the march and of the dance equally show that accurate time-keeping gives a mysterious pleasure to the mind. Those who have been in the East have had proofs of this fact, by witnessing masses of men standing hours in the enjoyment of simultaneous movement; stepping backwards and forwards and right and left, keeping time to two or three rude notes on a whistle—the man who blows the whistle becoming equally intoxicated with the delight. Exact time-keeping must be regarded as a characteristic element of human pleasure. But it may be said that successive beats of a drum and equal time steps of a man are neither music nor beauty.

I say they are the first rude elements of it, and I will now proceed to show how out of these elements rich music grows. Take the single beat of a drum, and ask how that can be made a musical tone. In the hands of the accomplished drummer the roll of the drum becomes a musical tone. When men march two paces a second, and the drumstick beats four paces a second, the men march and keep time with pleasure; but the sound is not to the mind musical. When to the same time of marching the skilled drummer beats eight beats a second, the ear still distinguishes the eight beats and the mind counts them, and though there is a certain softness in these eight equal beats, there is measure, but still no music. But if the man has high skill, and gives sixteen beats in a second, which is eight beats for each man's step, the beats will begin to have a sort of musical charm. A hum will be heard in the air which, while it agrees with their march, has a pleasure sensation beyond it.

I will now take a second example, leading to the same point. I will take nothing that can be called a musical instrument. I will take the sound of my own footsteps as I march along. A flight of steps lies before me ascending from a lower to a higher terrace bounded by two walls. The avenue in which

I am walking is a wide one. When I mount the first step I hear a strange sound; it happens to be the exact sound of the musical drum. Each step I take my footfall is attended with the common noise and with this sweet sound, and all the way up the stairs this harmony continues; but when I come out on the broad platform above, it ceases. This strange result turns me back.

I descend the steps, and as I go down the musical sound accompanies me; but when I go out on the open platform beneath, the music has ceased. I now return, and mounting the steps once more I am startled by a new sound; a harmony of two sounds now accompany me, two notes which give me a great pleasure. Again, on arriving at the top, this ceases; but on descending I hear a new harmony quite different from what I heard before.

Now it will be admitted that my step on the ground has no title to be called musical vibration, nor an oscillation, nor a series of oscillations. The leather of a boot and the hard stone floor meet in a single stroke, which is not doubled or tripled or in any way repeated. What has happened is this.

Two upright walls on each side of the steps were 64 feet apart. I walked up the middle; the stroke of my foot on the flat stone sent out the intermediate air in a strong wave between my foot and the stone step, a wave which went right and left to the two walls, and reached the walls in $\frac{1}{32}$ of a second; each wave was sent back by the wall in the next $\frac{1}{32}$ part of a second, and thus both waves struck me at $\frac{1}{16}$ part of a second. Both waves having met in the middle passed me to opposite sides again, struck the walls, were again reflected from them, again came back, and all this happened sixteen times in one second before I put down my foot on the next step.

That next step made a new wave, which went out and came back exactly in the same time as the former, and this phenomenon was repeated sixteen times in the next second of time, when I again set down my foot for the third time, and created a third wave. Thus at each step I created a new wave, and that new wave by reflection struck my ear sixteen times in exact measure of time; and this exact measure of time gave to the unmusical sound of my step an equal measure of the $\frac{1}{16}$th part of a second, which repeated sixteen times in one second gave to my ear an agreeable sound.

But when I descended this flight of steps, I kept more to the right, being nearer to it than to the left in the proportion of two to one, and though my step and its noise were still the same, the sound conveyed to my ear was a sweet harmony of two sounds, both quite different from the former sound.

One was a higher, the other a lower musical sound. Their relation to the former sound was what musicians call a fifth, what we may call two to three in relation to the former sound; but in relation to each other, each gave one beat for the others' two, and so still kept time. In going up the second time, instead of keeping twice as near to the right, I kept four times as near, and so got a musical sound which musicians call a double octave, and also a new relation called by them a third, and which is really a relation of five to four, the distance at which I stood from the walls being $\frac{1}{5}$ on one side, and $\frac{4}{5}$ on the other.

What I wish to prove by this example is, that the original moving force, its accompanying noise, and the nature of the musical result produced on my ear belong to totally different physical phenomena, and may indeed be said to have nothing in common except this common cause. Also that here there was no oscillation or vibration, but a single source carrying a single force to the ear in successive number keeping exact time.

If, therefore, it be true that harmonious sound merely means harmonious time-keeping, that agreeing times measured by agreeing numbers are the cause of pleasing sensations in the ear which we call melody or music; if this be true, then, in order to create musical sound, we have only to provide the means of sending out successive equal time, beats, strokes, claps or other mechanical modes of creating solitary carrier waves, to transmit to the ear a series of impulses which, through the mechanism of the ear, give to the mind the pleasure of musical sound.

I will now give a few experimental examples, showing how musical sounds are created by simple strokes or shocks, each independent of the other, only keeping time.

Take a still night and a paved street. Listen to a carriage approaching from a distance. The timing of the horses' feet gives me an idea of the pace at which they are going. I happen to know that the street is paved with granite stones, and that those stones are laid in rows edgeways, so that each row of stones occupy exactly 6 inches along the road.

As the carriage wheels roll along at a rapid pace of ten miles an hour, there comes to me from the distance a rolling sound resembling the rolling sound of a huge organ-pipe of very low pitch. As it comes nearer and nearer the noise grows louder and louder; but the pace continuing the same, I hear exactly the same tone at the same pitch as the organ-pipe.

What I hear is merely the stroke which the carriage wheel gives to each successive stone as it leaps to the next, and at 10 miles an hour, which is 16 feet a second, it delivers thirty-two strokes on thirty-two separate stones in each second of time; and one second of time thus marked off into thirty-two equal portions, each sending off one wave into the ear, gives me the same sound as if thirty-two beats were sent into a 32 feet organ-pipe, and sent out from it into my ear.

A very delicate ear will discover a very curious change in the musical quality of the sound after the carriage has passed. The loudness, as it approached, gradually increased, and the loudness, as it goes away, will in like manner gradually diminish; but it will notice that the quality of the sound has changed sensibly.

There must surely, then, have been some change in time to make this change in tone. Can the vehicle be going slower? but this is antedated by the equal pace of the horses. The cause is found in the fact that the vehicle and the sound wave were *both* in the first instance travelling towards the ear, whereas, after passing, the carriage was going forwards away from the ear, while the sound was still travelling toward the ear.

The sound, in the first instance, as it drove towards us, had both the speed of the carriage and of the travelling wave; but when it had passed, the vehicle was travelling one way and the sound another.

Thus into one second of time, when approaching, thirty-two beats were crowded, while in going away there were thirty-three, a difference to a delicate ear quite appreciable.

Thus a division of time of delivery without change in the origin of sound has given to the ear a difference of musical tone. But the more rapid and uniform speed of railways enables us to test the nature of sounds still more accurately. On railways in England these experiments are not so easily made as on some foreign lines, where long lengths quite straight and level along smooth plains enable us to make uniform observations. Conceive such an even smooth line to have on one side a wooden fence made of upright posts set one foot apart.

The train going twenty miles an hour passes sixty-four of these posts in each second of time. The careful listener hears a curious hum, which he may know to be the sound of C from a 16 feet organ-pipe, and which he can easily test if he has in his pocket an ordinary tuning-fork. As his ear passes each post the sound of the train striking the post is sent back to the passing ear, and each separate post sends into it its own reflection of the train noise. The noises made by the train may be many, various, and incongruous, and anything but musical. Nevertheless, sixty-four wooden posts passed by in one second of time give to the ear, in exact time, measured by the uniform speed, sixty-four strokes in one second, and the result is the continuous musical tone of a 16 feet organ-pipe.

By and by the train comes to a wide fence, where the posts are 2 feet apart, and now, though nothing else is changed, there are only thirty-two posts passed in a second, and the tone is now that which the same organ-pipe would give out if its top were closed instead of left open. The musical effect is an octave lower than it was before, because there is now one post in place of two.

We will make a third experiment. A train is approaching a stopping station. The fence is now as formerly one foot apart, and the posts send sixty-four beats in the second. Slowing, the tone descends in pitch, and when the train comes down from twenty to ten miles an hour we only pass thirty-two posts in a second, and the tone descends an octave. Thus the same effect has been produced on the ear by slackening the train as was formerly produced by widening the space between the posts.

In England I have heard a like sound from a like cause, not as a passenger in the train, but as a passer-by in a field. There is a fence common in Kent made of split oak in thin vertical boards, not laid flush side by side, but tilted over each other, so as to break the continuity of surface at each joint. Watching the passing train from a considerable distance, I hear the tone of the organ-pipe distinctly and continuously given, only of the much higher pitch corresponding to the narrow breadth of the boards.

The next example is perhaps also the most instructive. A loud bell rings as a passenger train approaches a station. Suddenly the bell appears to have changed its tone. That tone was E by my tuning-fork, and it suddenly lowers to D, that is to say, it was lowered by one-tenth part; and from this we may draw the conclusion that the train was then travelling at the rate of one-twentieth of the speed of sound, or thirty miles an hour.

I think I am justified in saying, that the test of measured sound is equal timed impulse delivered into the air and conveyed by it in equal timed separate waves into the ear, and that the symmetric nature of this mechanical effect gives to the mind its sensation of pleasure, and thus symmetry in time is to the ear what symmetry in form is to the eye.

Musical Tone.

Adopting the conclusion thus tested that agreeable sound arises from exactly timed impulse sent by successive detached waves, and by them delivered into the ear, we shall be enabled clearly to perceive the origin and understand the nature of what we call tone in musical scale. It is astonishing how few in number and how simple in nature are the tones which form the elements out of which all musical compositions are formed. They are almost as simple as the world which the ancients created out of four elements, which four elements in modern science are called solid, liquid, gaseous, and ethereal.

The four musical numbers are 7, 8, 9, 10, and these four, which the ancients called a tetrad, and which we might call a quatrain, we have doubled and called an octave. To understand the tetrad, and to derive from it the modern octave, we must form clear ideas of certain groups of numbers, and we must contrive that the sounds we wish to create shall be formed of exact groups of these numbers, and that these numbers shall all represent exact times.

We will start with the number 7. This is the number from which all tones called F are made, and as there are three F's in a human voice and a greater number in the organ, we have to distinguish the F's into degrees, which we can do by planting degrees of numbers on the right shoulders of the letter F thus—

$$F^1, F^2, F^3.$$

According to this notation, F^1 means a tone given by 7 beats a second, but that is to ordinary ears inaudible. F^2 means a tone given out by 14 beats a second, and that is barely audible to the educated ear. F^3 means 21 beats, which is audible, but still very low. F^4 is 28 beats a second, a low bass tone; and F^{10} is 70 beats, which is an ordinary tone of the human voice. Each of these is said in musical language to be an octave higher than the other, and they are all only multiples of the number 7.

We have taken the number 7 as the first of the tetrad. We will now take the number 10 as the last, and call it B. The first B^1 being 10, the second B^2 is 20, the third B^3 is 30, the tenth B^{10} 100—each of these sounds being created by that number of beats or waves in a second. The lowest degrees in this tone are, like the former, inaudible to ordinary ears, and the highest is the ordinary tone of the human voice.

Between these extremes of the tetrad come the numbers 8 and 9. These numbers are the elements of the tones G and A. G is caused by 8 beats in a second, and A by 9 beats. The successive degrees of

G are 8, 16, 32, 64, and so on; and the degrees of A are 9, 18, 36, 72, 144, &c.

These four give a perfect tetrad, such as we have in the bass of an organ-pipe, provided we give them all in the same degree. Thus $F^6 = 6 \times 7 = 42$. $G^6 = 6 \times 8 = 48$. $A^6 = 6 \times 9 = 54$. $B^6 = 6 \times 10 = 60$. All in the sixth degree.

We will now pass from this perfect tetrad to the octave. We found the tetrad by making 7 our starting point, and placing the numbers 8, 9, 10 above it. To form the octave we have only to repeat these numbers *below* 7, but each of them in a *lower* degree. We have selected F^6, G^6, A^6, B^6, in the sixth degree, and keeping them there we now place the same numbers, 8, 9, 10 in the fourth degree, and thus get $C = 4 \times 8 = 32$, $D = 4 \times 9 = 36$, and $E = 4 \times 10$ 40.

Thus, by merely arranging the same elementary numbers in due symmetric order above and below our standard number (7) of the tetrad, we get the true octave, which we now represent as follows:—

Tetrad.

F^6 G^6 A^6 B^6

C^4 D^4 E^4

Num. 8, 9, 10, 7 8, 9, 10. Elements.

4 × 8	4 × 9	4 × 10	6 × 7	6 × 8	6 × 9	6 × 10	
32	36	40	42	48	54	60	beats equal timed.
C	D	E	F	G	A	B	

Octave.

Taking the central number 7 F we have formed one tetrad to the right by combining with it 8, 9, 10, and a second tetrad to the left by combining with it the same numbers, with this difference, that on the right we have placed the higher degree and on the left the lower degree, and the consequence is, that we have now got the melodious sequence of seven tones, miscalled the musical octave.

One other consequence of this symmetric arrangement is, that the three tones on the right and the three tones on the left are in exquisite harmony; the C on the left is in harmony with the G on the right, because they are both members of the family of eight, only in different degrees. The D on the left is in harmony with the A on the right, because they both belong to the family of nine, only differing in degree. E on the left harmonises with B on the right, because both belong to the family of ten in different degrees.

ON MUSICAL NOMENCLATURE.

Musicians have adopted a system of nomenclature in regard to musical tones which is both obscure and inaccurate as regards expression of the different qualities of sound, and which, if it does not convey ideas which are absolutely false, does certainly not assist either in clear thinking or accurate expressions. They call the difference between one tone and another an interval. They call the difference between successive notes equal intervals of a tone each, and where this is not true they call it a semi-tone, and when this becomes untrue they are obliged to talk of a bigger and a smaller semi-tone or a major and a minor semitone. Thus the measures they use are the following:

One tone.
A half tone.
A bigger half tone.
A less half tone.

But the absolute truth is, that the interval between one note in the scale and another stand to each other in quite other relations than the numbers 1 and 2. The first tone above the standard 7 is higher by three beats than the 7 which has twenty-one, whereas the note lower than the 7 differs from it by one beat; therefore the interval below the standard 7 is one-third of the interval above it. In this case, therefore, the true expression of the relations between these intervals would be that one was one-third lower than 7, while the other was three-thirds higher than 7.

In this way we should clearly express the relations of the intervals if we said that the interval between F and G was three-thirds of a tone higher, and that between F and E was one-third of a tone lower.

Between E and D there are two-thirds of a tone. We have thus an absolute relation of successive intervals of 1, 2, 3, and in the entire (so-called) octave we have none other than these three intervals, Writing out the octave we should have the following numbers as representing the true intervals :—

THREE FUNDAMENTAL OCTAVES.

$$\underbrace{8\quad 9\quad 10\quad 7\quad 8\quad 9\quad 10,}_{2\qquad F\qquad 3}\quad \underbrace{8\quad 9\quad 10\quad 7\quad 8\quad 9\quad 10,}_{4\qquad F\qquad 6}\quad \underbrace{8\quad 9\quad 10\quad 7\quad 8\quad 9\quad 10}_{8\qquad F\qquad 12}$$

21 42 84

16 18 20 24 27 30 32 36 40 48 54 60 64 72 80 96 108 120

INTERVALS. INTERVALS. INTERVALS. INTERVALS.

2	2	1	3	3	3	2	4	4	2	6	6	6	4	8	8	4	12	12	12
C | D	D | E	E | F	F | G	G | A	A | B	B | C	C | D	D | E	E | F	F | G	G | A	A | B	B | C	C | D	D | E	E | F	F | G	G | A	A | B
↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓
$\frac{2}{18}$	$\frac{2}{20}$	$\frac{1}{21}$	$\frac{3}{21}$	$\frac{3}{24}$	$\frac{3}{27}$	$\frac{2}{30}$	$\frac{4}{36}$	$\frac{4}{40}$	$\frac{2}{42}$	$\frac{6}{42}$	$\frac{6}{48}$	$\frac{6}{54}$	$\frac{6}{60}$	$\frac{8}{72}$	$\frac{8}{80}$	$\frac{4}{84}$	$\frac{12}{84}$	$\frac{12}{96}$	$\frac{12}{108}$
=	=	=	=	=	=	=	=	=	=	=	=	=	=	=	=	=	=	=	=
$\frac{1}{9}$	$\frac{1}{10}$	$\frac{1}{21}$	$\frac{3}{21}$	$\frac{1}{8}$	$\frac{1}{9}$	$\frac{1}{10}$	$\frac{1}{9}$	$\frac{1}{10}$	$\frac{1}{21}$	$\frac{3}{21}$	$\frac{1}{8}$	$\frac{1}{9}$	$\frac{1}{15}$	$\frac{1}{9}$	$\frac{1}{10}$	$\frac{1}{21}$	$\frac{3}{21}$	$\frac{1}{8}$	$\frac{1}{9}$

C 2 D 2 E 1 F 3 G 3 A 3 B 2 C 2 D 2 E 1 F 3 G 3 A 3 B 2 C 2 D 2 E 1 F 3 G 3 A 3 B 2

This table shows three fundamental octaves compared together in tetrads according to the method I have adopted, and then compared together by the method of intervals arranged in the same order. The conclusions to which this table leads are the following:—

The upper series of numbers, consisting of 8, 9, 10, arranged above and below 7, give us three entire octaves without further complications, excepting that each ascending tetrad is multiplied by 2 and 3 alternately, and that each higher octave has double the number of the lower octave. Thus, the first octave is multiplied by 2 and 3; the second by 4 and 6; the third by 8 and 12. We have thus got in the upper half of the table the characteristic numbers of all the tones of three octaves in lucid simplicity.

In the lower half of the table I have endeavoured to reduce the method of estimating tones by their intervals to the utmost simplicity of which it is capable, and the result is, that the relations of the intervals are such that the numbers 1, 2, and 3 are predominant, and that tones and semitones do not and cannot truly express the relations either of a complete tetrad or of a complete octave, all the intervals being in the proportion of 1, 2, 3.

B, C, D, and E represent, or have between them, intervals of 2.

E and F have between them the interval of 1.

F, G, A, B have between them intervals of 3.

That an octave, therefore, can be represented as consisting of eight notes, having between them intervals of five equal tones and two semitones, must be nonsense. If intervals are to be considered at all as characteristic of tones, they must be considered as made up of one smallest interval, which we may call the unit, one larger interval or twice the unit, and one largest interval or thrice the unit.

I should consider it a great advance both in the art and science of music if we would cease to represent the relations of sounds by intervals, and would represent them by the characteristic numbers which indicate the number of separate waves in each tone, and if we would associate with this number the successive degrees of the same tone.

Let us try how by this means we may proceed to create, develop, and distinguish all possible musical tones.

As the tone F is formed by seven waves repeated over and over the higher it rises in pitch, why not call it *tone seven?*

G being formed of eight waves becomes *tone eight.*

A is formed of nine waves, and therefore may be called *tone nine.*

B is formed of ten waves, and can be called *tone ten.*

The notes 8, 9, 10 above (7), and the notes, 8, 9, 10 below (7), would bear to each other the exact relation now called a perfect fifth—a phrase which is quite misleading.

If in this way tones should be represented by numbers, we must consider how all third relations can also be represented by numbers, and truly represented in form.

Taking as the representative of an octave the number 2, we must distinguish an octave higher from an octave lower, and this might be done in the simplest way by placing them as follows:—

An octave = 2.
An octave higher = $\bar{2}$.
An octave lower = $\underline{2}$.

The next relation in music is that which is commonly called "a fifth," but as the tone itself has no relation to the number 5, and is only truly represented by the number 3, the designation is misleading. The true relation at present called fifth and octave is only the relation of the numbers 3 and 2; it would be wise, therefore, to represent a fifth by the number 3, and its relation to the octave by the number 3 to 2.

A fifth = 3 to 2.
A fifth higher = $\bar{3}$ to 2.
A fifth lower = $\underline{3}$ to 2.

A third is the equally misleading term given to another relation in music, which can only be accurately described by the number 5. The relation of a third means, in truth, that one tone is making five vibrations at the time when the other tone called the octave is making four, and the actual mode of representation of the third in music would be the number 5, employed as follows:—

A third = 5.
A third higher = $\bar{5}$ to 4.
A third lower = $\underline{5}$ to 4.

These three relations, the octave—the third and the fifth are the three relations which we call perfect harmony. They are called the perfect chords or perfect harmonies, because every second vibration in the one tone agrees with every *third* vibration in the tone above, with every *fourth* vibration in the tone above that, and with every *fifth* in the tone above that.

It should be noticed in this relation that the tone four is merely the tone two doubled or an octave higher.

The other musical relations represented by the numbers 7, 8, 9, 10, are the melodic relations of music as distinguished from the harmonies. 7, 8, 9, 10 are an agreeable succession of sounds, as they form a symmetric variety, and in going downwards from 7 10, 9, 8 form also another though different variety. In order, therefore, thoroughly to understand musical relations both in melody and harmony, we must first master the relations of the numbers 2, 3, 4 and 5 for harmony and then the relations of 7, 8, 9, 10, and of 7, 10, 9, 8 for melody.

HARMONY.

II. III. V.

Two waves agreeing with *one* make
An "octave" (I. with II.)
An octave higher = $\overline{2}$.
An octave lower = $\frac{1}{2}$.

Three waves agreeing with two make
A "fifth" (II. with III.)
A fifth higher = $\frac{3}{2}$.
A fifth lower = $\frac{2}{3}$.

Five waves agreeing with four make
A "third" (IV. with V.)
A third higher = $\frac{5}{4}$.
A third lower = $\frac{4}{5}$.

MELODY.

The Central Tone.

Lower tetrad.			7	Higher tetrad.		
8	9	10		8	9	10
Modified.				Modified.		
16	18	20		24	27	30
			21			

Their common bond,

$$\frac{16}{21} \quad \frac{18}{21} \quad \frac{20}{21} \quad \frac{21}{21} \quad \frac{24}{21} \quad \frac{27}{21} \quad \frac{30}{21}$$

Their next differences,

$$\frac{2}{21} \quad \frac{2}{21} \quad \frac{1}{21} \qquad \frac{3}{21} \quad \frac{3}{21} \quad \frac{3}{21}$$

These melodic relations *grow out of one central or standard tone,* which grows out of the number 7. This number 7 may be modified into different octaves by the number 2, or into fifths by the number 3, and thus it will be changed into 14 beats (an octave higher, or into 21 beats, a third octave higher than the first).

We shall find 21 the most convenient modification of 7 as a standard of comparison with the other melodic tones.

We will now take the other melodic tones.

Lower tetrad.				Upper tetrad.		
8	9	10	7	8	9	10

To obtain variety let us modify the upper tetrad by 3, and the lower tetrad by 2. This converts the upper tetrad into 24, 27, 30, and the lower into 16, 18, 20.

These two groups on either side of 7^3 make up the following melodic series of Seven of the musical scale :—

7

modified by 3 into

21.

$$\begin{array}{lllll} & 8^3 & 9^3 & 10^3 & \text{modified by } 3 \\ \text{into} & 24 & 27 & 30. & \\ \hline & 8^2 & 9^2 & 10^2 & \text{modified by } 2 \\ \text{into} & 16 & 18 & 20. & \end{array}$$

Then set in melodic order

16 18 20 21 24 27 30.

Thus we have obtained from 8, 9, 10, modified in 2, groups 2 and 3, the complete melodic scale.

Differences and Discords.

In melodic series we have found a mutual relation of the numbers 8, 9, 10, to 7, which, when duly modified, united them in one melodic scale.

The nature of each tone in this melodic scale we can most conveniently and accurately express in the numbers showing the relation of the waves in each tone to the waves in the standard tone. This is done by the following series :—

$$\tfrac{16}{21} \quad \tfrac{18}{21} \quad \tfrac{20}{21} \quad 21 \quad \tfrac{24}{21} \quad \tfrac{27}{21} \quad \tfrac{30}{21}$$

This series enables us to show how each step in this scale differs from or agrees with each other step. The first and second step starting from the left differ by 2, the second and third step differ by 2, but the third and fourth differ by 1, while the fourth and fifth differ by 3, and the two following also differ by 3.

Thus we find that by taking the standard 21 waves of the central tone (7^3), and measuring the differences between the tones by this standard, we find three distinct differences in the scale, represented by the numbers 1, 2, 3. It is this diversity of difference between tones in the same scale which creates discord and leads to confusion when not properly understood, and which has been misrepresented by the system of miscalled semitones.

The following are successive differences in the tones of the scale as measured by the common standard of 21:—

Differences in the number of Waves between two successive tones.

Taking the tones of the standard scale as measured by third standard number, we have

16	18	20	21	24	27	30

and we have as their differences

2	2	1	3	3	3

It is plain that the differences between each tone and its next neighbour are far from alike; they differ as the numbers 1, 2, 3, so that if we call the largest difference that of a tone, the next difference will be that of two-thirds of a tone, and the next one-third of a tone.

The common language of music misrepresents this difference by calling the lesser a semitone, or half of the larger tone, and when the error of this is perceived it is corrected by inventing two new errors—

a major semitone and a minor semitone, which leads to endless confusion.

The true differences between two tones of the melodic scale are exactly expressed thus :—

1, One tone = Difference = $\frac{3}{21}$.
$\frac{1}{3}$, One-third tone = Difference = $\frac{1}{21}$.
$\frac{2}{3}$, Two-thirds tone = Difference = $\frac{2}{21}$.

Beginning with the smallest difference which exists between one note and another, and calling *that one unit* our *standard unit of interval,* the intervals of an entire octave would be as follows :—

Tone.	Interval.
C to D	2 units.
D to E	2 "
E to F	1 "
F to G	3 "
G to A	3 "
A to B	3 "
B to C	2 "
	16 "

Adding these together, we find that the entire interval between the extremes of an octave consists of sixteen units, and it was probably that fact which misled musicians into supposing that an octave consisted of sixteen intervals that were all alike, and that, therefore, each of them might be said to consist of two semitones, whereas, from one octave to the next octave, there are only seven intervals, made up of sixteen units.

We are now in a position to distinguish the tones which will agree from those which will disagree, in any given melodic succession and in any given simultaneous harmony. We will take a series of tones extending from sixteen to thirty-two vibrations —16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32.

We will choose out of these what will form a melodic series, and strike out those that will not. Starting with sixteen, we will leave out 17, because we want a difference of two units between the first and the second. This brings us to 18, as the second note of the series. The same reason obliges us to strike out 19, as we should not have the interval that we desire. Thus 20 would be the third interval, 21 remains intact, while 22 and 23 both disappear, and 24 is the next note in the scale. We also want an interval of three units for our next note, and therefore 25 and 26 are struck out, and 27 remains. In like manner, three units are wanted for our next interval, and therefore 28 and 29 are struck out, and 30 remains. 30 is the last note of our octave, but it is not the last interval, because the next octave must begin with the double of 16, which is 32. We therefore strike out 31.

TABLE.

16	18	20	21	24	27	30	32

Why we want in melody these successive unequal steps must remain apparently without reason, unless we enter upon the field of harmony, where an adequate explanation is found with certainty and clearness; but before we leave the question of melody, it is interesting to know that any deviations or fallings away from this standard reveals a strange and exceptional mood of the mind, which, whenever it is stricken with grief or melancholy, expresses that feeling by deviating from this standard, and afterwards when it returns to content, or rises to joy, expresses its condition by a return to the melodious standard. This expression of grief is called a minor key.

To pass on to the explanation of this scale of melody, from the laws of harmony we must go back to the origin of harmony in sound. The harmony of sound is the same as the harmony of numbers. The number 8 agrees with 32, because every step in the one agrees with every second step in the other. 16 agrees with 24, because every second step in the one agrees with every third step in the other. For the same reason 18 and 27 agree with each other. In like manner, if the note C is sounded at the same time with its octave, every second beat of the air on the ear keeps time with one of the others. If C^{16} is sounded along with G^{24}, every third beat of the one keeps time with every second beat of the other, and if E^{20} is sounded with G^{24}, every fourth beat in one keeps time with every fifth in the other. Thus the numbers which represent sound in harmony, when played together, represent the same sounds which, when played successively, give the pleasing sensation of melody. Although between 20 and 21 E and F there is but one unit, and therefore an entire discord, yet it is forced upon us because it has the definite relation to 18 and 27, which binds them together by groups of 3. It is also bound by 3 to 24 and 30, but in a much less degree.

The message carried by the Sound Wave and delivered into the Ear.

The message from a distant source has now arrived at the portal of the ear by a wave travelling at the rate of 1000 feet a second. If the messenger be still strong and fresh he rushes into the open porch and goes straight forward until stopped by a screen forming a division through which he cannot pass. All that the carrier wave can do is to knock at this screen, which yields gently to every push given, whether slight or strong, without either giving way or admitting the messenger. The message, however, is delivered while the messenger is excluded. Behind this screen is an inner hall or audience chamber where the message is received. From the roof of this chamber is suspended what we may call a knocker, which receives through the screen the successive beats, knocks, or impulses which the messenger delivers. These beats or impulses were delivered into the audience chamber in numbers 2, 3, 5, 7 in given time, in different degrees of loudness, arranged in various groups and in the same order and intervals in which they were sent out. Within the audience chamber we find a mechanism for further transmission.

A machine made of various levers, joints, cranks, and links has been set in motion by the knocker, and this machine has to do the following work. The audience chamber is like the outer hall or portal filled with air and furnished with appropriate means of ventilation. Beyond this chamber is a mysterious labyrinth, and what comes of the messages delivered into that labyrinth we must endeavour to conceive.

The work which the mechanism in the audience chamber has to perform is very difficult, because it has to move particles 800 times heavier than the matter which gives it motion. It received its impulse from the air, and it has now to propagate it through a dense fluid or minute lake filled with thousands of minute atomic mechanisms arranged to receive the impulse, and these mechanisms are arranged in geometrical order along the walls of seemingly endless spiral chambers, conveying to us the same idea of multitude and complexity which we gather from the inside of a cathedral organ when we see it for the first time. In fact this labyrinth chamber is but the cathedral organ of which the two outer chambers are the nave and transept, and all the ingenious keys, stops, pedals, reed-pipes only do on a large scale the combined evolutions which are here performed on an infinitesimal scale.

We will now consider how the ear is fitted for the reception of the solitary wave.

When the ocean tide wave reaches the mouth of a river, it becomes modified in speed, size, shape, by a multitude of circumstances in the shape of the coast, the conformation of the bottom, and the width, depth, and length of the channel which receives it. Some conformations of a coast throw off the tide wave and send it away. Other conformations favour its propagation. Some channels break it to pieces; others utilise it to the utmost by economising its force, removing obstacles out of its way, accelerating its speed and concentrating its efforts on points to be attained. All such like efforts have to be accomplished, and are accomplished, by the mechanism of the ear;

and in order to know how this is done, it is necessary first to get rid of some erroneous theories founded on the belief that sound waves are oscillating waves instead of the solitary wave of translation.

The radical difference between these two sets of phenomena is nowhere more strongly shown than when we consider the effect of *shape* upon sound. By shape is here meant the shape of the channel through which a sound wave travels, or the enclosures which confine it, and between or along which it has to move. When a hearing or speaking trumpet or a musical trumpet are described, we are told that its virtues consist in the reflection which takes place from one side to its opposite side, and it is supposed that the striking of a sound first on one side, then on the other, and so on over and over again, has something to do with the accumulation of the sound. It is not so. This may concentrate noise; it would only confuse hearing.

What really happens is what happens to the water wave in like circumstances. When a solitary wave strikes on a flat surface at a large angle it is completely reflected off from it, just as an oscillating wave is reflected from the side of a pond or a ray of light from a mirror. But when a solitary wave encounters a smooth wall, not set right across it but alongside its path, and only differing from it a little, this side resistance exercises a curious modifying effect upon it, the result of which is, that the wave alters its course, leaves off its own direction, and takes exactly and truly a new direction parallel to that of the wall. Now, this is an important fact hitherto overlooked or unknown in the phenomena of sound, and let us next trace its immediate consequence. Imagine that instead of one wall on one side a wave has two walls on either side, and imagine these two walls gently coming nearer and nearer as the waves go further and further. What must happen in the end? As the wave moves along between the two gradually approaching enclosures, if it were reflected from one side to the other we should immediately see a maze of confusion—a single wave being broken up into a thousand little waves all crossing each other. But the fact is quite the contrary. We see the solitary wave entering the wide channel, going straight on, and, instead of being dispersed or broken up, become more decided and well marked in shape. Its crest rises higher, and it goes forward quicker instead of slower. It remains a solitary wave, more compact in size, more marked in shape, and possesses increased unity and energy.

It is on this principle that the hearing and speaking trumpet receive sound, and on which the musical trumpet acts when it gives out its various sounds. The ear itself, with its channels and chambers made so as to guide the carrier wave without reflecting or breaking its course, is an exact exemplification of the same principle.

One of the most remarkable phenomena which occurs in the transmission of each solitary sound wave from the performing musician to the listener, is that which takes place in the audience chamber beyond the membrane. The wave in the outer porch is an air wave, but the wave which has to be generated in the innermost chamber is a water wave (or nerve wave, or mysterious wave of mind and matter).

Now the manner in which this is done is a marvellously precise characteristic of the solitary wave. The first difficulty to be overcome is, that an air wave has to be employed to generate a water wave, and as air is only $\frac{1}{1000}$ part of the weight of water, it requires ingenious contrivance and complex machinery to accomplish it. The joints, levers, and cranks by which it is accomplished are accordingly admirable specimens of hydraulic machinery, which I will endeavour to make clear.

At the back of the entrance hall of the ear and in front of the audience chamber is a thin elastic stretched membrane, which excludes the air, but is flexible enough to receive the wave impulse, and by bulging inward conveys the force to any object in contact with it. Pressing close upon its centre is one end of a lever made of bone, the other end of which is suspended from the roof of the audience chamber. The membrane end of this lever is rounded so as to form a joint by which it can pivot freely in obedience to the moving power on the membrane. A second lever also of bone hangs from the top of this chamber, and the two levers are so shaped as to fit exactly into each other, and form a freely revolving joint at their union. These two levers meet at a point of mutual pressure, where the motion of the first is communicated to the second, and the long arm of the second, which descends from the roof, thus receives powerful pressure, which it communicates to a third instrument. The third instrument is a piston of an oval shape with a double piston-rod in a bent shape (which has given to it the name of stirrup), and this bent rod is placed at right angles to the end of the other lever, so that an impulse from it is sent horizontally *inward*. This piston exerts a strong pressure on the inner wall of the audience chamber, in an opening in which it is inserted.

Thus the push given to the lower end of the first lever passes (through the medium of the second) on to the piston in contact with the inner wall, and in this arrangement there is a design worthy of special notice. This mechanism can convey a push forward, but it cannot convey a pull backwards; it can convey an impulse, but it cannot *convey an oscillation*. The mechanism of the ear offers absolute contradiction to this. It is perfect for conveying each single solitary wave, but incapable of conveying the oscillatory or vibratory waves which are attributed to it.

We have now reached the inner wall of the audience chamber beyond which the mysterious chamber begins, into which we have not yet intruded. No air can enter this chamber; it is excluded by being closed and quite filled with a watery fluid. Two windows are, however, left,—one looking into the audience chamber. This opening is made exactly to fit the oval piston; a thin elastic membrane fills this oval window, and by the pressure of the water bulges outwards; into this exactly fits the oval piston. When therefore a single wave of air pushes (by means of the second lever) the piston through the window in the water chamber, it thus produces the water wave which starts on its travels through the mysterious labyrinth.

Just as the tide sends forward the bodies floating on its surface, just as it gives motion to the rivers into which it passes, just as it picks up the stones on a beach, throwing them forward on each other, and as it sends forward the sands of a bank and the weeds of an inland shore by a regulated and measured motion, so does this microscopical tidal wave move and remove, lift and lower, strike and stir all those marvellous varieties of mechanism which are placed in countless numbers on the shore of the water chamber to receive them.

The spiral form of the mysterious labyrinth performs an important part in the nervous or mental analysis of sound. Without inquiring into the nature of nervous energy, we can at least gather from the spiral arrangement and the geometric proportions of the labyrinth what are some of the functions it has to perform. Let us each suppose himself placed in this nerve centre. Our duty there will be to receive in very short time a multitude of messages and to interpret and mentally dispose of them in an intelligent manner. What could be more convenient than that they should be placed in a continuous row all round the centre, than that those of one sort should lie towards the right, and those of an opposite sort towards the left. Those on the right might be the longest messages and those on the left the shorter, and they might be so methodically arranged as to follow a graduated scale. Now in this circle there are places easily remembered: first, there are the four quarters of the circle; then there are the thirty-two points of the circle, and then, more minutely, the 360°, all easily comprehensible. Thus messages of 360 kinds might be distributed under the notice of the central observer.

But going a step further, we might conceive that the messages are increased in number and variety far beyond the provision we have made. We have then to seek for a method by which a circle that is already completed shall start afresh on a further extension without breach of continuity. Now this we find is exactly supplied by the method of the conic spiral. The original circle is preserved unbroken, and is uniformly prolonged by raising its level on the extreme left uniformly and gently above the original level, so that when the first circle is completed a second circle begins, goes on, and is completed all round the same centre, but rising always gently upwards at the same rate.

Now this process of continuing a circular platform round a central axis on a gently rising incline may be continued upwards by a third circle, or, if needful, by more; but it is worthy of observation that in the three circles we should now have got 1080° in three stages of 360° each; in other words, 1080 different communications are now symmetrically arranged under the mental eye of the observer in a method capable of easy and accurate discrimination. If more minute and multitudinous discriminations were wanted, it would only be necessary to divide each degree into three parts of twenty minutes each in order to have provided 3240 separate discriminate places.

Something analogous to all this is what is provided in the labyrinthine chamber, but how that is connected with the different sounds of music, and what relation can exist between a continuous spiral platform and the tones of a voice or of a violin string, is by no means self-evident. In order to discover how this spiral platform can assist in the discrimination of musical sound, we must first analyse the musical relations of sound, and then find out how these can be represented on a spiral platform.

The Geometric Relations of Musical Sound.

We have all been charmed with the beauty of the curves of the harp. Those curves were forced upon its constructor by the fact that in no other way could he obtain within moderate limits of space the necessary variety of musical sound, and he found that within the limits of the sounds required to accompany the human voice the gently winding curve of the harp (which has in it a certain hyperbolic nature) gave him all he required. But the extreme range of musical thought, of which the mind is capable, and the extreme range of the nature of sound, require a far wider means of discrimination than the curve of the harp can supply. When we consider the range of sound which exists between the tone of a 64 feet organ-pipe and the chirp of a grasshopper, we cover at once the whole 1080° which the three circuits of spiral platform comprehend.

To connect the sounds of music with their geometric representation in the inside of the ear, let us begin by conceiving a long level line going from right to left, and divided into twelve equal portions, each an inch long. Let us draw an upright line an inch long at the starting-point, and let us divide it into sixteen parts, and if we imagine each of these parts subdivided into four parts, we shall have a line divided into sixty-four parts. We may take this as the representative of an organ-pipe 64 feet long, and that we may take as representing the lowest audible musical sound. It should be remembered that this sound is the note C in the musical scale, and is produced by a mechanical cause or impulse repeated sixteen times in a second. It may be called the tone due to sixteen waves a second. Leaving this starting-point, let us go an inch further to the left, and there we erect a second upright line half the length of the first, and therefore representing an organ-pipe 32 feet high, which gives out thirty-two blows or waves a second, or twice the number of the previous pipe. Now this doubling the number of waves indicates a sound which we call an octave higher than the first. We have now, therefore, got a second C. Going to the third stage, we erect a new upright line, half the height of its predecessor. It represents an organ-pipe 16 feet high, which gives out sixty-four waves in a second instead of thirty-two, and the sound it gives out is called C, and is an octave higher than the previous pipe, and two octaves higher than the first. At the next stage we place a vertical line representing an organ-pipe 8 feet high; it gives out 128 impulses or waves a second, and its sound is C, three octaves higher than the first.

C^6	C^5	C^4	C^3	C^2	C^1
512	256	128	64	32	16
C^{12}	C^{11}	C^{10}	C^9	C^8	C^7
32,768	16,384	8,192	4,096	2,048	1,024

Thus we have risen between the second stage and the twelfth from 32 waves in a second to 32,768 waves in a second. These twelve stages we may regard as representing and measuring accurately and covering the whole range of audible sounds.

If from these largest intervals of sound, called octaves, we descend into their subdivisions, into the various degrees of harmony and the minute steps of melodic scale, we have only to divide the horizontal line in a single stage into sixteen equal divisions, and proceeding from the right place to make an upright line at 2, a second at 4, a third at 5, a sixth at 8, the next at 11, and to proportion each of these upright lines in height according to the following numbers:—

16 18 20 21 24 27 30 32.

This long straight geometric scale is, however, of a size and shape quite unsuited to the human ear. To make it suitable for our purpose, we must make it leave its rectilinear form, and wind round a central axis, and when we have done this and made it perform two circuits with the number of octaves we have got, we shall find that we have transformed the straight into the spiral without any change in the proportion of the lines which represent the length of the organ-pipes for the musical tones. We shall find that each octave covers 60° of the circle, and that the same radial lines which divide the inner smaller circle equally divide the large circle into the like number of divisions, so that, on ascending the platform from the entrance of the labyrinth, and going round two turns or two stories of the spiral, we shall have covered the whole ground of musical sound, and given a true picture or map all round the central axis where the auditory nerves are concentrated, or where we have supposed ourselves to be. The result will be that, if we place in each of the positions thus marked out a delicate sensitive organ like a minute violin string or a minute nerve filament capable of responding to a musical tone, then, when any piece of music on the organ at a distance is performed, each sound wave delivered into the ear will communicate its message to the corresponding recipient in this spiral scale, and the sensitive mind in the centre will recognise the particular tone in the two circular scales which each wave of sound indicates, and the music without will be converted into mental music within.

PART III.

ON THE GREAT OCEAN OF ETHER, AND ITS RELATION TO MATTER.

Empty Space.

GIVEN, something able to hold fast on one settled place in space.

Call this thing a material point. Give this thing a *power* to keep all other things out of that place, and to occupy a certain room all round it.

Call that power repulsion. Measure that power by the amount of space kept clear, and that amount of space one atom's sphere.

Call that one central atom hydrogen. Give that atom power to draw other like atoms towards it all round about it—each preserving its own sphere, no one intruding on the other sphere.

Take another point in space. Place in it a new thing. Give that new thing power to keep other new things at a certain distance, let that distance be only one half of the former distance, call that thing oxygen. Group eight atoms of this in one space; each of these eight atoms will have its own sphere, but the whole space occupied by the group will only be equal to half the sphere of the hydrogen atom and its surrounding sphere.

Hydrogen and Oxygen combined.

Take one atom of hydrogen in its sphere.

Take eight atoms of oxygen, each in its own sphere, but clustered in one group by their innate power of attraction. By some external force pull these eight atoms asunder far enough to get the atom of hydrogen and its sphere into the midst of their group. The eight atoms of oxygen with their sphere will still cling to one another, and form a chain or enclosure round the sphere of hydrogen held fast in their midst. This joint group is one atom of water, but to effect this change the work to be done by the eight atom group of oxygen is to squeeze the double sphere of hydrogen into the bulk of what we will call one standard sphere. The effect of this squeezing is to throw out what I will for the moment call *heat*.

Spheres around the Atoms.

This question now arises, What is it that occupies all the void space around each atom? It is important to know that the bulk of the atom of hydrogen is not $\frac{1}{100}$ part of the space it fills. Nevertheless it has power enough to keep all other atoms out of that space, and we must ascertain whether that space is empty or whether some ethereal element fills it up, just as the air fills up the space in a room.

If there is a sphere of infinitely refined air which is invisible and intangible, we must give it some fancy name like weightless fluid, or ether. We must either suppose that there are void spaces in the universe, or agree that what has been called a vacuum is only bulk occupied by ether, which cannot be touched, seen, weighed, or measured, but which fills all space.

We will now therefore make this inquiry, when the double spheres round one atom of hydrogen are added to the standard sphere, containing the eight atoms of oxygen, and when as the result a compound of nine atoms is made, constituting a particle of water occupying only two atoms' spheres instead of three, what has become of the ether which occupied the third sphere? Has it been forced out of its imprisoned enclosure through the squeezing of the oxygen atoms, and being thus cast out where shall it go? Answer—It must wait till it finds another atom centre waiting for its ethereal sphere, but it may now become the etherial surrounding, not of a hydrogen atom or a group of oxygen atoms, but of carbon or nitrogen atoms that show a special attraction for it.

Thus we see that the compressed ether which in forced confinement struggled to be free and to escape, and by these efforts raised a high temperature, has flowed out into ether. I will term this flowing ether heat.

The Four Elements.

We have thus before us the following ideal picture: —A central spot, one atom in that centre, a sphere all round it, out of which that atom keeps others away. This enclosed space thus left free is filled with ether. What can be known of this ether?

A prime law proclaimed throughout the universe is that nature abhors a void. To fill the universal void or make it non-existing, the universe is filled with an ethereal matter having this quality, that it is always ready to yield place to any other matter ready to take that place. It must also have penetrating force to enter any opening, however small, and it must yield to pressure and flow out when the space it occupies is wanted. We find in it, then, these three qualities—pushing, penetrating, and flowing.

Thus the central atom has the power to displace the ether and occupy its place. It has also, like the ether, power to push other atoms to a certain distance, leaving it in the centre of a sphere of ether, and it has the additional power to draw from a distance atoms like itself similarly surrounded by an ether sphere. Atoms thus surrounded and symmetrically arranged and distributed become the elements of geometric organisation.

To illustrate this I will return to the one atom of hydrogen with its double sphere imprisoned by the eight atoms of oxygen, each in its own small sphere of $\frac{1}{8}$ of one standard sphere. These eight are connected with each other by strong attractive bonds, by which they form a prison or cell enclosing and finally squeezing the sphere of the hydrogen into half the bulk.

We must try to imagine how each of these eight atoms with their spheres are placed apart from and in relation to each other.

1st. They are all at the same distance from the centre atom of hydrogen.

2nd. They are each sundered from their nearest neighbour by the same distance.

3rd. They are placed (as it were) in the corners of a square box containing a round ball. The twelve straight lines which form the edges of the square box measure the distance of the eight atoms from each other and each atom has got three neighbours with whom it forms a group of four. These nine atoms thus symmetrically arranged make one particle of water.

To maintain each atom in its due distance from the other atoms the attraction and repulsion inherent in them are so proportioned that at the farther distance the attraction exceeds the repulsion and at the nearer distance the repulsion exceeds the attraction.

This is done in accordance with two laws.

I. The law of Newton, that attraction diminishes with the second degree of the distance.

II. A new law, that repulsion increases with the third degree of the nearness.

At a fixed distance these two forces balance. This distance is the radius of the atom's sphere.

If two atoms were alike in attractive force but somewhat different in repulsive force, the atom with the greater repulsive force would occupy the centre of a larger sphere. If this force were such as to keep an atom at a distance of ten times its own diameter, while another kind of atom had the power to keep its like at twenty times that distance, then the one would be surrounded by a sphere eight times as large as the other, and a group of any number of the one would contain eight times as many atoms as the other.

If therefore to each atom is given a certain measure of repulsive force and also a certain other measure of attractive force, then each variety of combination will constitute a new kind of matter, and the relations of these atoms will grow out of the numbers which measure these forces. The number two to one represents hydrogen and oxygen as repulsive agents, and eight to one represents oxygen and hydrogen as attractive agents, and one, two, and eight are the measures of hydrogen and oxygen. These numbers when added, 1, 2, 8, 16, 9, 18, represent water :—

Repulsion.		Attraction.
2 Atoms' spheres Hydrogen	. .	1 Atom Hydrogen.
+ 1 Atom sphere Oxygen	. .	+ 8 Atoms Oxygen.
= 2 Atoms' spheres Water	. .	= 9 Atoms Water.

In this chemical combination of oxygen and hydrogen to form water, the resulting water is in the form of water, gas, or steam. The bulk of ether expelled is equal to one sphere standard.

The water gas or steam occupies no more bulk than the hydrogen did before combination, or, it occupies twice the bulk the hydrogen did before combination. Its weight is nine units in two standard spheres, and it has given out one sphere of ether flowing out in heat.

To make this *water gas* into water or into ice, the atoms it contains must be brought twelve times nearer to each other than they now are. They will thus occupy the $\frac{1}{1728}$ part of their present bulk, and all the ether in this space will be driven out, excepting so much as can find room in the $\frac{1}{1728}$ part left.

We will select some new atoms called carbon and take the number 6 as representing carbon natures. We will place one atom of carbon in the middle on the bottom, one in the middle of each end, and two on the middle of the front and back of an imaginary square box. There remains one to be set in the middle of the top before the box is closed down, and now the six atoms of carbon are symmetrically arranged in the same box with the one atom of hydrogen and the eight atoms of oxygen, and in these positions the attractive and repulsive forces of all the atoms tend to keep each other firmly in place. This hydrogen-oxygen box full of carbon is called carbonic oxide.

Nitric oxide would give us more trouble, because its group of atoms is seven, being one more than we have room for. I should therefore have to turn out the atom of hydrogen from its place in the centre, and then, by putting six nitrogen atoms in the place of the six carbon atoms, and one in the place of the hydrogen atom, we should get an oxy-nitrogen compound.

These four elements constitute the great bulk of the world of life around us. The air we breathe is oxygen and nitrogen. The nitrogen we breathe out finds its way into the plants which want to take it in. The water we drink, like the ocean and the clouds, are merely hydrogen and oxygen united, and in a thunder-storm the force we call electric, clashes hydrogen and oxygen together, sends out the heat of the lightning flash, and sends down the combined atoms in the round form of water drops or in the crystallised form of snow or hail. All trees and vegetables are almost entirely made up of these four elements, hydrogen, oxygen, carbon, and nitrogen, and to convert these into the matter of which all animals are made, we take the elements of that same vegetable matter and add to them a small quantity of one or more of the sixty other substances which as far as we know comprehend the whole universe, including suns and planets and every living thing upon them.

But returning to our four elementary matters, we must not be surprised to find that it is in relation to them that the others have much or any importance. Oxygen and hydrogen form the casket in which we place some one of the sixty minerals, and each of these sixty minerals gives to the wide world, filled by the four elements, a special use, a colour, an odour, a taste, a weight, and some of the infinite variety of qualities which form the countless materials of worlds such as ours.

Relation between the Ether and other Matters.

The first stage in the creation of a world formed according to law and order would be a relation between the ether which fills all space and the atoms set down in it in separate portions, forming centres, and drawing by their inherent force of attraction like atoms towards them. Although the ether can exercise no sensible attractive force upon the atom or atoms, owing to the absence of mass and weight, it may itself be subject to an action from the atoms, in the same way as the earth attracts to it a dense atmosphere of air, which atmosphere, a few miles off, becomes nearly imponderable, so, within a certain sphere, all around the atom the ether may be drawn close to it, crowded in, and stored up. Make a group of atoms, say four, gathered round a common centre, the four would exercise four times more influence on the ether than the one, and within this space would be crowded the ether which surrounds each atom separately; whilst, by the concentration of their force, this ether would be crowded in towards the common centre, and would form there an accumulation of ether, densest at the centre.

Outside the Cell: Inside the Atom Sphere.

We have been examining the complex constitution of the elementary matter of the world in the simplest state in which we find it. First, a solitary atom called hydrogen settled in the centre of a vast space 3000 times larger than it fills, and out of this space it has power to prevent any like atom from intruding. But there is no need for such intrusion, as each like atom is also set in the centre of a like sphere, with a like power of repelling intruders, and each atom is thus kept at a distance of nearly thirty times its own diameter from its neighbour. In these circumstances only some outside compulsory force can crowd the atoms closer by crushing the spheres into smaller bulk, and as these spheres are highly elastic they yield to this crushing force, and afterwards recoil to their first bulk.

When we took the more complicated particle of oxygen divisible into eight separate atoms, and grouped them together in a geometrical form, we found them settled in an atom's sphere only half the bulk of that of the hydrogen, and when we enclosed by force the hydrogen in the oxygen cells we also compressed the triple spheres into a double sphere. In each of these cases the bulk of the central groups is less than $\frac{1}{1000}$th part of the enclosing atmosphere, and even when we add other substances to the central group they occupy but a minute portion of the whole.

For the present we shall treat each sphere as an unoccupied bulk, and consider how it can be crowded in such a manner as to resist any outside force tending to crush or compress it. There are two ways in which we could conceive this to be done. If the atoms were enclosed with a pushing force strong enough to keep each of their neighbours at their present distance, the sphere between them might be considered as an absolute void, and every communication between one atom and another would be impossible; but as we know that no void exists, we have taken the universal ether endowed with this repulsion as the medium of communication between the atoms and as the filling force of each atom's sphere.

Let us consider what relation can exist between this ether and its atoms. We know that atoms attract each other, and it is impossible to doubt that just as the earth attracts her atmosphere, so each atom attracts its sphere; and as we know that the material attraction grows strongest with the square of the nearness, so the attraction of each atom will be felt by its ether to be 1000 times stronger close to the atom than it is at the outer edge of the sphere. Thus close to each atom its own attraction will give us a condensed sphere, just as round the earth the air is densest on the surface.

What change will now take place in the condition of two atoms' spheres, when by force their central atoms are united and two spheres are crowded into one? Eight atoms of oxygen will possess eight times the attractive force of one atom of hydrogen. The number eight is the cube of the number two. If, therefore, this eightfold force were introduced into the centre of the double sphere of hydrogen, this force would be sufficient to crowd that double sphere into one, and so reduce these three spheres into two, which is exactly what happens.

Let us next endeavour to crowd these ethereal spheres by external force instead of internal. To do this we must first provide a box or cell, and the cover of that box must be so moveable that it can slide downwards, fitting so exactly that there is no escape for the ether. At each step downwards the ether will resist more and more strongly, and half-way down there will be twice as much ether in a given space as before, and the ether will pass outwards with a double force.

Instead of this square box and sliding cover, I will take a flexible bag impenetrable to ether; and if I have external force like an hydraulic press to squeeze that bag (which is full of ether) into half its bulk, the pressure of the ether outwards will be doubled; and if I continue increasing pressure until the bulk of the bag is reduced to an eighth part, the resisting force of the ether will be eightfold, and the bag will be half its diameter.

Thus by external force I have done the same work as was before done by the attraction of the atoms. It is necessary to distinguish carefully between the opposite nature of these two operations, and the identity of the effects produced.

We can next conceive a case in which these opposite ways might be combined in jointly producing the same effect, by attraction from within, and compression from without. Conceive that I have placed in the same bag the hydrogen and the oxygen spheres; conceive that I crowd the two into one by force, and that the atoms crowded together are brought so near that the attraction of the eight oxygen draws in and encloses the hydrogen atom. The three spheres are enclosed in the bulk of two, but there are still within the enclosure the three original spheres of ether, and the consequence is that when pressure is removed the attraction of the oxygen will still hold fast. The atom of hydrogen and its sphere in the reduced bulk, and the surplus sphere of ether released from pressure will rush out, and this flowing out ether will form a unit of heat. Whether therefore we act by compressing force from without, or by attractive force from within, we must remember that the actions upon the ether are inevitable, and of a quite different nature from the actions of the atoms. The two are mixed together, but must not be confounded as they often are.

We thus see how in certain combinations of atoms the concentration of attractive forces, crowding their respective spheres into less bulk, has the effect of either condensing the ether, or leaving it to flow out into the surrounding space, forming a flow of heat. Let us now fancy a reverse operation, and that atoms and atom's-spheres have been already crowded; that the ether has already escaped, and that now some rival forces should be drawing the combined atoms away from each other in opposite directions. Conceive that the confined hydrogen atom is released from the grasp of the eight oxygen by the greater force. The released hydrogen will now want to resume its double atmospheric bulk, but there will now be a void space within for all that ether which originally filled the double sphere. Whence can that ether be brought? It has escaped into the surrounding atmosphere, and the present process is taking place beyond its reach.

Where then can this wanted ether be found? It can only be got by borrowing from its near neighbours. If their spheres are full, some of that ether will by its repulsive nature flow into the atmospheric void, and gradually fill it up. Meanwhile there will be intense local cold which will subsist in and around the sphere until ether has been borrowed from its next neighbour, who again borrows from his next, and so on, in turn spreading the void over a wider and wider space with continually diminishing intensity.

Thus it is that in many chemical combinations the increased attractions of the atoms brought near each other produce the effect of crowding them into diminished bulk, and sending out a large excess of ether into surrounding space, and so, as we say, causing heat, as is always done in the process called combustion, which is merely a process of releasing ether from confinement to flow freely out. On the other hand, the creation of cold by what are called freezing mixtures is only the release of confined atoms into larger spheres which there is not ether enough in the released combinations to fill up, and there is a strong flow of the ether out of all the neighbouring cells which causes what we call cold in them and all about them. Some such changes in the volumes of ether take place in varying degrees in all chemical changes in Nature and in Art.

The Atom's-Sphere and its Heat.

An atom's-sphere that will hold much ether is said to have a great capacity for heat. An atom's-sphere which has been enlarged by the entrance of the ether from without is said to have a high temperature.

What the total quantity of ether may be in one atom of matter, forming its specific heat, it may be impossible to know, because we cannot empty any space of all its ether and then measure how much will fill it again. All we can do is this: We take hydrogen when water is freezing round it, and we take the same hydrogen when surrounded by the steam of boiling water, and measure how much larger it is in bulk in the latter case than in the former. The increase of ether we call the measure of heat between freezing and boiling point, or 100°. These 100° we call centigrade.

I beg to be permitted to make a digression on the language in use regarding heat, cold, and temperature, which language is apt to be misunderstood, and therefore to be misleading.

The bottom of the scale or freezing-point is called zero, and marked 0. The top of the scale is called boiling-point, and marked 100°, and each step of the scale should mark a uniform proportioned increase of bulk. If it did so it would be called a perfect scale. Common scales are generally imperfect as measures of quantity, the cause of the inaccuracy being that uniform scales of equal length are not uniform scales of growing bulk.

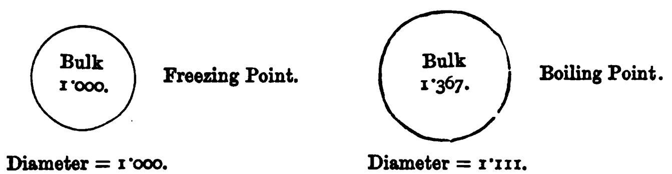
The word zero of scale, which means the bulk of hydrogen when water around is freezing, is misleading when it is applied to a quite other zero which exists in words, but which we are quite unacquainted with in fact. *The absolute zero of heat is a fiction* *convenient for assisting certain modes of thought:* it arises from the natural attempt to go below freezing point, and ask whether that bulk of hydrogen, which has diminished from boiling down to freezing point, might not be further diminished 100° lower, and then called 100° of cold. We should then have one scale going up, which we call 100° of heat, and another scale going down, which we call 100° of cold, thus getting 200° of temperature.

But to this process there is no end; one might continue to repeat figures of cold downwards to 200, 300, or any other number of degrees, just as one finds in fact that we have heat and flame giving heat and growing bulk, to the extent of 200°, 300°, 400°, 500°, which is the heat of red-hot iron, and 1000°, which is the heat of bright flame.

Downwards it is believed we cannot travel far; 73° is the farthest we can rationally reach, and we sometimes call this point the *absolute zero.* It is a convenient, wise term, indicating the absolute starting-point of all true scales of heat; but neither the number 273, nor any other number short of infinity, truly represents its relation to matter and bulk.

The absolute zero of heat has no just and true meaning but one. It is the point in space at which bulk is reduced to nothing, and from which its growth starts.

I will now start from two known facts of heat, the bulk of an atom sphere of hydrogen at boiling-point, and its bulk at freezing-point. Fortunately these two bulks are accurately known. Calling one 1000 measures of bulk, the other is 1367. If we take these two bulks in the spherical form, the smaller represented by a globe one inch in diameter, the larger by a globe 1·11 inches, we shall have the means on which to found true scales by heat and cold.



In this representation of growing bulk in spherical volume we have a true representation of actually correct scales of heat. We have only to take a succession of spheres growing larger to the right and smaller to the left, always in the same proportion, and we shall obtain an absolute and just scale of heat, with an accurate zero on the extreme left indicated but never reached; and on the right an infinite and ever-growing scale of heat.

While the sphere round a central atom has been enlarged or diminished by the entrance or exit of ether, causing heat or cold, the atom or the atoms' group in the centre must be affected by the surrounding changes. If it be a single atom, as hydrogen in its double sphere, it may be affected in one manner, if it be a compound group of one hydrogen amidst eight oxygen atoms, it will be affected in a different manner. We will consider the second condition first.

Starting from 100°, and the bulk of sphere 1·367, the central group may possibly not occupy more than the $\frac{1}{1000}$ part of that bulk, or

0·001367.

But if we diminish the sphere to 1·000, it might be supposed that the atomic group might close together in the same proportions, and so become

0·001.

We have now to examine whether this central group of atoms grows or diminishes in the same manner and by the same means as its surrounding sphere, or whether the central atoms follow a quite different law. For this purpose we will take the group of oxygen and hydrogen in the state of water, when the great bulk of their spheres has been removed, and when the atoms are so close as to be probably incapable of being compressed to half their present distance from each other by any known force. We know by direct experiment that the change produced on the atom's sphere, between freezing and boiling point, is from 1·000 to 1·367, but the change on the atom's group is from 1000 to 1050.

In other words, while the atom's sphere undergoes an increase of 367, the atom's group undergoes an increase of only 50. In round figures, the changes of bulk in the atoms between freezing and boiling point is an increase of 5 per cent., while on the sphere it is 35 per cent.

Experiments have also shown that this rate of increase remains nearly unchanged throughout some ten or fourteen stages upwards, and it is only in the fourteenth stage upwards that the distance grows to double, while in the same fourteenth stage the atom's-sphere would have grown to 2000. We will examine the conclusions to which this important difference must lead. It is plain that the atoms in the group must exercise a powerful attractive force not only on each other, but on the ether close upon them, and also upon the ether within the group, so that it requires seven times as much force to separate the atoms of one group from each other as it requires to separate a group from its neighbour.

This fact is most important, as showing the hidden cause of many of those phenomena which take place in steam boilers in that phase when a violent struggle is going on in the contest as to whether the atoms shall keep the state of water or assume that of steam; in other words, whether each group of atoms shall remain confined in its concentrated state as a drop of water, or become dispersed into an atmosphere of steam.

Heat in Relation to Ether.

Our investigations have now conducted us up to the difficult question, what is heat, and the plainest answer to this question is, that heat is not a thing, but rather a state of things, an event. Let us place together two objects, say two cannon balls, one too large to enter the mouth of the cannon, the other entering it easily. We will let them stand near each other for five minutes, and we shall then find that both can enter the cannon easily. Now leave them together for five minutes more, and we shall find that the ball which first entered easily cannot now enter at all, and that which could not enter, does so more easily than the other did at first, and we have here a physical fact demanding careful investigation.

We must proceed by careful measures by testing the size of each ball in an accurately shaped steel ring. The result will be, that the first ball was $\frac{2}{1000}$ of an inch larger in diameter than the second, therefore it could not enter, while the second could. Five minutes later we measure again and find that both can enter freely, the one having increased by the $\frac{1}{1000}$ of an inch, and the other having diminished by the $\frac{1}{1000}$ of an inch. We again leave them together for five minutes, and find by exact measure that the first has diminished by $\frac{2}{1000}$ of an inch, while the second has increased by $\frac{2}{1000}$ of an inch. What is the cause of this mysterious change? Ball No. 1 has been lying out in the cold in frosty weather. Ball No. 2 had been lying near a stove. No. 2 was in fact the smaller of the two, but had become the larger. When they lay together it first became the same size and then the smaller of the two.

We are driven to form some conclusion. We may say one of the balls has become cold and shrunk, and one has grown hot and swelled, or something has gone out of the one and entered into the other, and so changed them both from greater to less, and from less to greater; but these conjectures bring us no nearer the truth, only that we may call this unknown cause ether, and we may say that this ether has got into ball No. 1 and swelled it out, has gone out of it and left it shrunk, and has gone into No. 2 and swelled it out; it may also be interesting to know that nothing we could do could induce that hidden cause to go out of No. 2 into No. 1 and swell it out again, so that we may say so much ether has been spent, used up, and cannot be restored.

"Yes," the artillerist will say, "there is one way in which it could be restored." He will take No. 2 cannon ball, place it in the cannon's mouth, fire it off, and place No. 1 in the line of fire so that No. 2 shall clash directly upon it when it is moving through the air at high speed. At the moment of clashing the ether formerly sent out will rush back into No. 1 out of No. 2 and restore it to its original bulk; but it is worthy of observation that this restoration of its original power into the ball has been accomplished only by the expenditure of far greater power in the shape of gunpowder.

We can thus by accurate measure verify the inflow and outflow of ether from one mass of matter to another in contact with it, but we have another mode of transference of a quite different nature from that of two cannon balls lying side by side. I mean transference by wave motion from the sun. If I had brought bright sunshine on a cannon ball through a lens of glass or a metal reflector, all the events of its receiving in ether and swelling and then giving out ether and diminishing would have taken place in like degree and order, and ether would have been first stored up, would then have flowed out or been driven out by force. In this way the sun's force would have been expended in sending a wave through the ethereal element travelling at the rate of 170,000 miles a second. The force spent by the sun in creating this wave clashing with this enormous speed upon the iron ball, would send into it just that quantity of ether which it had before received from the stove.

Laws Governing the Forces of Attraction and Repulsion.

Science enables us to say that there are only four states of matter known to exist—the solid, the liquid, the aerial, and the ethereal, and it uses its function to discover the laws and forces under the influence of which, in these four states, all the phenomena of the universe are going on.

In order, therefore, to study nature as a whole, we should begin with the study of universal law; but the limited nature of the human mind cannot pursue clearly a multitude of different thoughts at once, and it is necessary to divide our progress into successive steps, and to direct our attention to one thing at a time. The best method of doing this is to take the example of a typical thing and a comprehensive thought. I have, therefore, taken the laws of attraction and repulsion as the two most comprehensive attributes of matter in number, order, and form.

The law of attraction, which unites matter to matter, being the most simple and most pregnant of the forces in nature, we must try to comprehend it on the ordinary scale of common life before thinking of it as dominant in the heavens and among the stars. We must think of it as binding each atom to another, and try to comprehend it as the same force, which by the same law governs the planets at millions of miles distance, and atoms at millionths of millionths of an inch apart.

In our own nearest plane of observation everything great and small, heavy and light, is drawn towards the earth, and falls towards it through 16 feet measure in one second of time. That is the measure all round the globe, and it is also the measure at the moon after due allowance has been made for the effect of distance in diminution of attractive force. This uniformity is called the law of gravitation.

In common life there are many things that would make us doubt this law. A piece of lead falls faster through 16 feet than a bundle of feathers of the same weight, and a balloon full of hydrogen goes up instead of down. It was these contradictory facts which kept the world so long in ignorance of this law; but we now know that when we make a vacuum in a chamber, a piece of lead, a feather, and a water bubble filled with oxygen, all fall down through 16 feet in one second of time. This 16 feet we must always carry in our minds as the standard measure of attraction; other effects of this fall we have next to consider.

A ball of iron dropped 16 feet, weighing 1 lb., and being in diameter 16 inches, strikes the earth a heavy blow. This blow we will measure by the rate of speed at which the ball was going when it struck. That speed was 32 feet a second. We must now reconcile these two different numbers. What the ball actually did we find by dividing the second of time in which it fell into four equal parts. In the first quarter it only fell 1 foot; in the second quarter, 3 feet; in the third, it fell 5 feet; and in the fourth, 7 feet. These four falls, when added, make the full fall of 16 feet. The reason of this disproportion is found in the fact that at starting the ball has no speed at all: we have therefore to think out the nature of a uniform growing speed, but to do this we must take a longer time and a greater fall, or two seconds of time instead of one. Let us think of these two seconds as made of eight quarters, and study the results of actual space fallen through in each quarter.

1st quarter	1 foot fall.	
2nd "	3 feet "	
3rd "	5 " "	
4th "	7 "	16 feet fall.
5th "	9 feet fall.	
6th "	11 " "	
7th "	13 " "	
8th "	15 " "	
Total,	64 feet.	

Speed and Space.

We must distinguish the nature of speed from that of space. The space fallen through in the first second is 16 feet, but in the second second it was 48 feet. The speed at the end of the first second was 32 feet, but at the end of the second it was 64 feet. Speed of fall and space traversed follow therefore a different law. Speed is doubled but space quadrupled in two seconds. What are these laws? Let us ask Nature what she would do in the third second?

In the third second of time divided into quarters we have:—

9th quarter		17 feet fall.
10th ,,		19 ,, ,,
11th ,,		21 ,, ,,
12th ,,		23 ,, ,,
		80
	Add,	64
	Total,	144 feet in three seconds.

The law thus obtained for speed in relation to time is :—

1st second	32 feet.
2nd ,,	64 ,,
3rd ,,	96 ,,

In other words, at starting there is no speed, at the end of a second it has grown to 32 feet. By that we do not mean that the ball has gone 32 feet, for that would be nonsense ; it has only gone 16 feet. What we do mean is, that it has somehow got in it the power to go on at the rate of 32 feet in a second if let alone.

How can we know that it has this power, seeing that it has not done it? If we catch the ball at the bottom of the 16 feet and turn it aside, so that it can roll along a level table, we find that it runs by measure 32 feet, or nearly a metre in each $\frac{1}{10}$th part of a second. This is the standard speed due to gravity. We have thus two measures of force—one we call distance or space, and the other speed. The force of gravity has a falling power of 16 feet, but it has a speed-creating power of 32 feet in a second.

Law.

There are not only two measures, but two laws. The first law is called "the law of the times;" the second is called "the law of the square of the times." The law of the times would be represented by the following numbers :—

2, 3, 4, 5, 6, 7, 8, 9, 10, 11.

The law of the square of the times by the following row of numbers :—

1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121.

Result of the first Law.

I.

Speed Created in Matter by Free Fall in Seconds of Time.

Speed Created.	Seconds of Time.
* 32 feet a second.	Created in one second fall.
† 64 " "	" two seconds "
‡ 96 " "	" three " "
128 " "	" four " "
§ 160 " "	" five " "
‖ 1600 " "	" fifty " "

* 20 miles an hour	=	Racing speed.
† 40 " "	=	Railway fast train.
‡ 60 " "	=	Express speed.
§ 100 " "	=	Hurricane speed.
‖ 1000 " "	=	Cannon ball speed.

Spaces fallen through in Seconds of Time.

In one second	16 feet fall		=	16	feet space.
" two seconds	16 "	× 4	=	64	" "
" three "	16 "	× 9	=	144	" "
" four "	16 "	× 16	=	256	" "
" five "	16 "	× 25	=	400	" "
" ten "	16 "	× 100	=	1600	" "

The Bond between Space and Speed commonly called Height due to Velocity.

Speed is the rate at which a piece of matter changes its place. It is therefore something inherent in its nature, and if it is capable of changes, we must call it the condition of the matter. Speed therefore is the condition in which a mass of matter finds itself with reference to change of place. It finds itself changing places quickly or slowly, and has no power in itself either to increase or diminish its speed, and if that speed changes its rate we suspect a cause, we seek for it, and invariably find it, and find it outside and not inside the moving mass.

Gravitation is one of these outside causes. If a body like a comet were passing near the earth in going towards the sun, and were turned out of its path downward to the earth, we should seek the cause of this deviation in the nature of the earth's attraction, and if we found this gave us the exact measure of the body's change, we should say that the cause of change of speed lay in the attraction of the earth. We now arrive at this question. How can some one speed be created in a mass of matter? and how can it best be measured? The answer is, By letting that mass of matter fall right down through a certain height, and this we call the height due to the speed. Every existing speed has a special height due to it, and this applies equally to planets, cannon balls, arrows, atoms.

Let us seek a standard speed and call it the speed due to 12 inches. A ball falls 12 inches in a quarter of a second, and in that time it gains speed enough to continue travelling on for ever with a speed of 8 feet per second or five miles an hour. 12 inches is therefore the height due to the velocity of five miles an hour. Let us follow out the consequence of this one fact. I take an ivory ball which I want to roll at the rate of five miles an hour, and I provide for it a smooth level table. I have now to contrive to let it fall through 12 inches height in such a manner that it may roll along a level line. I shall find it running along this line 8 feet a second without any change, unless obstacles in its way, or the resistance of the air, bring it to rest.

We will seek another standard speed and call it the speed due to 16 feet. Allow the wheel of a carriage to roll down a height of 16 feet along a slope, steep or gentle: when it reaches the bottom it will roll along a level road with the speed of twenty miles an hour. Twenty miles an hour is therefore the speed due to the fall of 16 feet.

Possibly the most instructive fact in questions of the nature of speed is that the manner of fall has nothing to do with the resulting speed. A ball falling 12 inches may fall right down in a quarter of a second, or it may go gently down a slope of 4 feet long: in both cases, when it gets to the bottom, its speed is found to be five miles an hour. The manner of the path has nothing to do with the speed, we may make any variety we please in the manner of descents. We shall always find it due to the standard height. Whether we roll the ball down an incline of any degrees of steepness, or swing it by a string of any length, or transfer the speed out of itself into another ball, we shall get the same quantity of motion, or the same standard speed due to the standard height.

We will now take the speed with which we move round the centre of the earth. That is a speed of 1000 miles an hour. The standard height due to this speed (which is also the speed of a rapid cannon-ball) would be 40,000 feet. Let us further apply our knowledge of the laws which connect together matter, space, and speed, to examine the motions of the planetary and stellar spheres. If a quarter of a second suffice to create a speed of five miles an hour or eight feet a second, it would only require ten quarter seconds continuation of the same force to create a speed of fifty miles an hour, ten half seconds to produce a speed of 100 miles an hour, and ten whole seconds one of 200 miles an hour. If ten seconds or one minute force will suffice for a speed of 1200 miles an hour in a mass of any bulk, 1000 minutes (16 hours) would raise the speed to 1,200,000 miles an hour, a speed far exceeding that of the earth and many planets in their course round the sun, and 16 hours is the time during which the force-creating speed would require to act in its full energy in order to launch one of the greatest planets on its course round the sun, and at its present rate of speed.

These same laws enable us to calculate with an equal precision the forces in the minute intervals between the atoms of matter as we witness their action in the phenomena of matter.

We will now go downwards from the fall of one foot in the $\frac{1}{4}$ of a second into the minute motions of the atoms, but it will be convenient first to divide the English foot into the smaller division of $\frac{1}{8}$ of an inch. It is a measure which all Englishmen understand. It is the ninety-sixth part of a foot, and for our convenience we will call it the $\frac{1}{100}$ part, which will necessitate our afterwards correcting all our calculations by the addition of four per cent.

But *should a wise decimal foot be adopted in the future,* caution may become unnecessary. With this proviso, we may now say that a body falls 100 eighths of an inch in $\frac{1}{4}$ second, and in so falling has a speed cf 8 feet per second, which we will now call 800 eighths of an inch per second.

Measures of Minute Motion.

To measure the minute motions of atoms with precision, we must accustom ourselves to exact thinking in minute portions of time through minute distances, with the same precision which astronomers apply to the heavens. For this purpose we must start from the same standard. That standard is :—

1 foot fall—$\frac{1}{4}$ second time—8 feet speed.

In order to go down to a smaller scale we use the following proportion, harmonising with the one above :—

$\frac{1}{10}$ of an inch fall—$\frac{1}{44}$ of a second—$\frac{88}{10}$ inches speed.

Going a step further we have—

$\frac{1}{1000}$ of an inch fall—$\frac{1}{440}$ of a second—$\frac{88}{100}$ of an inch speed.

Hitherto we have kept our old standard, but it is desirable that we should be able to go farther than this, and to calculate what happens in the $\frac{1}{1000}$ part of a second by the same forces. For this purpose we put our standard measure into inches.

192 inches fall—1 second of time—384 inches speed.

This yields—

·0192 of an inch fall—$\frac{1}{100}$ part of a second—3·84 inches speed;

and the practical result of this in round numbers is this following standard of minute motion arising from the attraction of gravitation—

$\frac{1}{5000}$ inch space in $\frac{1}{1000}$ of a second $\frac{4}{10}$ of an inch speed.

It will be convenient to substitute the word instant of time for $\frac{1}{1000}$ of a second.

Proceeding from these data, we can at once calculate that in $\frac{1}{20}$ of an "instant" the standard force of gravity will produce $\frac{1}{2,000,000}$ of an inch fall,—$\frac{1}{50}$ of an inch speed, and therefore, if we now couple together the idea of a rise and a fall, as in a wave, through the $\frac{1}{2,000,000}$ of an inch, we shall have 10,000 such beats or springs in one second of time, and the result in speed attained in each beat will be thus $\frac{1}{50}$ part of an inch per second.

When we come to examine the minute motions which arise between the atoms of matter in such cases as when two ivory balls moving at the rate of 8 feet per second, going in opposite directions, meet and strike, we must have recourse to these minute standards of time and space. If we suppose the yielding of two balls to each other to take place through the $\frac{1}{5000}$ part of an inch during an instant, and the recoil to take place during the same time, the whole speed the recoil could make would be $\frac{4}{10}$ of an inch per second, whereas the speed of collision was 8 feet per second.

It is plain, therefore, that the force of recoil must be far greater than the force of gravity in order to send back the ball after collision with the same speed with which it came into collision. $\frac{4}{10}$ of an inch speed compared with 8 feet speed in the same time, is $\frac{4}{10}$ of an inch speed compared to 96 inches speed, which is exactly $\frac{1}{240}$ of the speed wanted. The conclusion we draw from this is, that to send the balls back again with the speed of 8 feet a second, the recoil made in the "instant" must take place by a force 240 times greater than the force of gravity, which is the weight of the ball. Hence we may say that the elastic forces of the ivory ball are measured by 240 times its own weight.

One example will show the practical value of these calculations. We know that in order to do its due work a cannon ball must be started by the action of gunpowder in the breach of the gun with a speed sufficient to travel forward an inch in the first "instant," and as we know that a force equal to the weight of the cannon ball would only give us $\frac{1}{5000}$ of an inch in an "instant," we learn that the force of the gunpowder must be 5000 times the weight of the ball in order to achieve the required speed in that first inch of its course, and we can thence go on to calculate how far it will go at each successive "instant" of its travel along the bore of the gun, and how fast it will be flying when it comes out of the cannon's mouth. Between the cannon ball, the planet, and the atom, there is only the difference of mass, time, and distance.

This example shows that, the more minute the range of motion is, the greater must be the force required to generate speed in the required time. We must also recollect how extremely minute is each individual atom, and that even in a gas its distance from its next neighbour may be twenty times its own diameter; and as these atoms are seldom large enough to be visible, we are obliged to conceive them as being less than the millionth of an inch in diameter, and their ranges of free motion in the extreme limit $\frac{1}{100}$ of an inch, so that to produce possible effects in those limits in an "instant" of time the atomic forces must be enormous, and measured by thousands of times the force of gravity. These forces, their natures and causes, we have now to seek, and our search must begin by following the nature of gravity into these minute spaces, and by searching in the antagonist and the compensating and controlling forces which it may there encounter, and the resulting phenomena which they will produce.

Measures of Atomic Force.

Since the minute phenomena of the atoms of matter take place in extremely short instants, and require enormous force for their execution, these forces being far greater than those which move a planet or a cannon ball, we must find out true methods of enabling us to appreciate them accurately by measuring them precisely. The title, given by Dr. Thomas Young to this measure of atomic force of each particular kind of matter, is "Its modulus, and this modulus is discovered in the following manner."

Take iron as our example. We want to know what force binds one atom of iron to another. We know that the force of gravity pulls an iron ball, two inches diameter, towards the centre of the earth with a force we call one pound weight, and we seek to know how many hundred or thousand times greater than this is the force by which one ball of iron would pull another similar ball towards it when both are infinitesimal in size and distance. The method of Young is to take a long wire or bar of iron, of which he knew the weight per foot, to fasten one end of this wire to the top of a high precipice, and then let the coil of wire go down by its own weight. If the precipice be high enough we find at last that the wire breaks at the top by the fact of its weight and length. He then tried to add on more wire in case it had broken by fault or accident, and found that he could not add to its length without its immediately giving way at the top. This number of feet is the modulus or ruling number characteristic of the atoms of iron.

To show the value of such a number, I will suppose the question to arise whether a suspension bridge could be built across the Falls of Niagara or across the Straits of Dover. An iron bar an inch and one-eighth in diameter and a yard long, weighs 10 lbs., 1728 yards make a mile of this iron bar, and if hung up by its end would carry its own weight, which is 17,280 lbs.; but if the length were doubled this weight would become 34,560 lbs.; and at this length (two miles) ordinary qualities of iron would break; 34,560 is therefore the measure of atomic attraction of this iron.

And now it will be seen what conclusions (as to the atomic conditions of matter) can be arrived at by the simple means this modulus gives. Iron and steel are among the hardest, strongest, toughest materials, and may be said to possess the quality of solidity as distinguished from fluidity in the highest degree; nevertheless if you will conceive a sea of iron formed of iron plates laid on the top of each other, the iron at the top would retain its ordinary condition, and would be solid as ice on the surface of a frozen lake; but at the depth of two miles the iron will have become fluid from this reason, that the attraction by which its particles are held together has been neutralised by the enormous antagonistic weight.

We thus measure the enormous forces at work in the case where a solid is changed into a fluid, and the reverse case of a fluid being changed into a solid.

Perhaps another illustration may be given of the extent of antagonistic forces called into play between atoms where water is turned into ice, or ice into water. To turn 1 lb. of water atoms into 1 lb. of ice, or 1 lb. of ice into the same weight of water, requires the pressure of a column of water 110,000 feet high; or if this is done by heat, the force of heat required is the force which would raise 1 lb. of water to the height of 110,000 feet. This 110,000 feet may be called the modulus of ice, or the depth of ice which will become water by its own weight.

Now that we have obtained through Young's modulus an exact standard measure of the forces with which the atoms of matter keep near to or away from each other, and with which they oppose the forces that pull them apart or crush them close, we can make a further step towards discovering the laws which bind the atoms together and keep them in place.

The difficulty we have to encounter is that of conceiving how atoms so small as less than a millionth of an inch in diameter could by any possibility attract each other so strongly as to support an iron chain two miles high hanging from a *single atom.*

To solve this we must start from a unit of a 1 lb. iron ball 2 inches in diameter or an inch radius, and from the unit of the earth's attraction for this iron ball. For this purpose two equal iron balls are hung from a ceiling by slender threads, and parted two inches asunder, with the object of seeing whether and how much they attract each other. It has been found that they do draw each other nearer together and do prevent each other from hanging plumb down; but the force is so small as to be only the millionth part of a pound weight. When put nearly close together the force is four times as great, so that it would require to be 250,000 times greater before the attraction of the two balls could support even a single pound's weight, instead of supporting two miles of iron chain.

Sir Isaac Newton's law that the attraction of gravitation increases not only with the nearness, but with the second power of the nearness, now comes to our assistance. When the two balls are quite close, so that they touch, the centre of each ball is still 1 inch from the touching point, and the two centres of the two balls are still 2 inches asunder; therefore although two single points of the ball are quite close, all the other atoms are far asunder, and this distance, according to Newton's law, enfeebles their attraction.

We will now seek to bring all the atoms of one ball much nearer to all those of the other. This can be done by beating out the ball into a thin plate of 1 foot in diameter; and this plate would be $\frac{1}{100}$ of an inch thick. If we hang these two plates by two threads from a ceiling quite close to each other, we shall find that they are powerfully attracted, and we can easily measure their attraction. The centres of the balls were 2 inches apart, and the plates are $\frac{1}{100}$ of an inch apart. By Newton's law the attraction is 40,000 times greater than it was before. This 40,000 is the duplicate of 2 inches compared with the $\frac{1}{100}$ of an inch, which is simply 200 to 1; and the square of this duplicate is 40,000 to 1.

Let us go a stage further with this, and beat out our plates into over a yard (38 inches) diameter. The plates when now hung up will be $\frac{1}{10}$ part of their former thickness, which will give a hundred times their former attraction, or 4,000,000 to 1.

We have already said that the diameter of each atom of iron is less than the millionth of an inch, and therefore, going a step further in this process, the attraction would become 800,000,000 to one, and it is plain that by continuing this process we should soon obtain for our one pound ball the attraction necessary to support a chain two miles long.

Force, Heat, Ether.

The enormous forces we have employed to overcome or neutralise the attraction of the minute atoms of matter to each other may have their work far more easily and conveniently done by the invisible agent called heat. If instead of a chain two miles long and 20 tons weight we employ half a pound of coal and a small blowpipe, we shall dissolve the whole of these powerful atomic forces and neutralise the 20 tons of adhesive power; we shall create so violent a repulsion between the atoms of iron that they tumultuously increase the space they occupy, and those particles which, closely locked together, formed a solid, now parted and free, have become a fluid. We have seen before that we could only turn the iron from a solid into a fluid by heaping upon it an enormous mass of iron two miles deep, in which case each atom would have been burdened with more than it could carry, would have lost all force of its own, and would have had to yield to any force sending it in any direction, depriving it of shape, order, place, till it would finally disappear.

What these gigantic forces do in their manner of action from without, the unseen agent heat does in its mode of action from within. Fuel contains this agent in a highly concentrated state, and under certain conditions it rushes out, pierces between the atoms of the iron, occupies all the spaces left between them, and, once entered, it swells out larger and larger, and by so doing increases the distance between the particles, and finally pushes them so far asunder that they can freely pass through and round one another, and thus become liquid.

Instead of measuring force by tons of weight, we have only to measure it now by degrees of heat. Our commonest measure of heat is "enough to make 1 lb. of water boil," and then we find that just ten or twelve times that quantity is enough to melt 1 lb. of iron. For convenience we call the quantity of heat which boils water 100, and the quantity which melts iron 1000. In like manner we measure all other degrees of heat by this standard, which the Latin race call "centigrade," and which we often also use as more convenient than our own standard with the inconvenient number of 180.

The comparison we are now able to make between the disruptive force of heat and that of 20 tons weight of iron (each acting on the minute atoms, which bind together an inch bar of iron) gives us an insight into the nature of this unseen agent. 1000° of heat can do as much work on 1 lb. of iron as an external crushing force of 20 tons. What is this invisible agent? A cubic inch of iron may seem to us to be already quite full of atoms of iron quite close to one another, but Sir Joseph Whitworth has found that he can compel a 12 inch bar to shorten itself into 10 inches, thus proving that 2 cubic inches of space were left void. Into these void spaces in the iron ether had already entered; there were therefore in the 12 cubic inches of solid iron 2 cubic inches of ether, and even after Sir Joseph Whitworth has done all he can, there still remains room for much greater reduction in bulk, by much greater external force.

Whether those void spaces are filled with ether in a solid condition, like a crystal cell, or in a liquid or an aerial condition, like water or steam, or whether it does not in itself constitute a new condition of force occupying space like or unlike any of the other elements, remains to be examined.

Ether—what we can know about this invisible element yet mighty force.

One thing we know about it is, that space is filled with it, that it fills every void, both where matter is and where it is not. We also know that it is an element of mighty force, and that it can transmit that force from space to space with great speed and without loss or waste. It can, moreover, use or transmit with the same speed the forces that it finds in atoms of matter.

Force and motion are then the specific properties of ether, and the nature of the force by which it works is repulsion. Whether it possesses within itself any attraction for itself, or whether it exercises attraction on other matter, we do not know, because if it does, this attraction is so small as to escape our means of measuring it.

What gives Ether its force.

Having concluded that ether occupies space and place, not only in what was called a vacuum, but also within those substances called gases, airs, liquids, and solids, and even occupies in them larger spaces than their own atoms, we may now proceed to the examination of the force by which it enters those spaces and keeps possession of them.

It is plain that the characteristic force of the all-pervading ether must be *power to push.* The attractive or *pulling* power which draws matter into shape, and enables it to keep shape, is wanting to it. Even the attraction of the earth seems lost upon ether, and cannot give it any sensible weight; for while we can weigh air, and measure specific weights of special kinds of air, we find no weight in ether, and can attribute to it no attractive force. Its characteristic is distractive force, or power to push, none to pull. This is its first law.

We will try to consider the nature of this all-pervading element. It is fluid—so that it can be divided into infinitely small portions without hindrance, yet, like water or air, offering great resistance to any force which should attempt to squeeze it into smaller bulk: refusing also to enlarge its bulk beyond that of its standard condition, just as water refuses to swell out in the manner air does.

Forces of Atoms in relation to Force of Ether.

Since the ethereal fluid, ether, has only one kind of force, namely, repulsion, or pushing force, while material atoms have two kinds of forces, we must recall to mind the relations between those twin forces in matter while investigating the effects the ethereal force will produce upon them. The law of attractive force in the atoms, in conformity with the law of Newton, is according to the *square of the nearness*, and I propose to take as the law of repulsive force, the *cube of the nearness.* I think I am justified in taking this as the true law of repulsion of atoms of matter, because I find from the researches of eminent chemists that all free gases do so expand as to double their bulk by an increase of the distance of the particles, in the ratio of the cube of their nearness, or as 111 cube to 367, and that the expansion and contraction of the atoms in their gaseous state, when compressed by external force, is in exact conformity with the same law.

The two laws of attraction or repulsion being settled as holding good for all kinds of matter, we have to introduce a modifying quantity for each different species of matter. This modifier of attractive force is called in Newton's law the mass, and is represented by a number modifying an assumed standard mass.

In a like manner I must introduce, along with my law of repulsion, a modifying number which I shall derive from a standard distance at which that repulsive force is exactly equal, in each kind of matter, to the attractive force in the opposite direction.

The three considerations, therefore, which may guide our calculations of the atoms are—

The law of attraction;
The law of repulsion;
The modifier.

Let us take an atom of hydrogen. We have seen that it occupies a standard bulk which it is convenient to indicate by the number II., while we indicate its mass by the number I. These may be called the characteristic numbers of hydrogen.

Let us now take an atom of oxygen. Its attractive force is measured by the number 8, while its repulsive force is measured by the number I.

Thus we arrive at the following relations between oxygen atoms and hydrogen atoms, that two hydrogen atoms are kept asunder by a repulsive force of 1·26, while two oxygen atoms are kept asunder by the force of I.

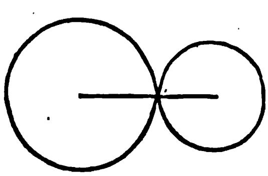
Let us now take these two standard atoms, oxygen 8, and hydrogen I., the former in its atom's-sphere I., the second in its atom's-sphere II.

Although II. is twice as large as I., it should be observed that the diameter of II. is only 1·26, the single globe of that size containing double the volume of the oxygen globe whose diameter is I. It should be remembered, therefore, that a globe which is double of another in the bulk it occupies or the room it contains, is only $1\frac{1}{4}$ times the diameter of the other. Thus a 15-inch globe would be twice as large as a 12-inch globe. This should be kept in mind, because an atom of oxygen and an atom of hydrogen alongside of each other, with their spheres touching, will have the centres of their atoms at the distance from each other of 2·26. We can now consider the relations of these two touching spheres to the great ether ocean round about them.

The atoms'-spheres represent the space kept empty by the repulsive force which each atom exercises on the other. Whenever, therefore, we attempt to force these atoms nearer, or to diminish the bulk between them, we shall have to encounter the pushing volume of the ether contained in each sphere.

This doctrine that each atom's-sphere is a *plenum* of ether will be found to play a most important part in the phenomena of physics and chemistry. We will therefore carefully examine the condition of each atom.

I have stated that one atom attracts another with the force of 1 and 2, and that they repel each other with a force that keeps them apart at a distance of I. on the right hand from the point of contact, and 1·26 on the left hand.

1·26  I.

Hydrogen. Oxygen.

Together these globes make three measures of ether, and before we could bring their atoms closer we should have first to remove the ether out of the way, and the question is how would the ether and the atoms behave when such an attempt was made.

The ether will oppose any attempt we make to dislodge it, and before effecting its dislodgement we should have to find a new vacant place ready to receive it; or we may try to cram it into smaller bulk, against which it may rebel.

The first result of the effort to crowd an atom's ether into less bulk, giving rise to a strong effort of opposition on the part of the ether, creates what we call *heat.* We may therefore define heat as *the effort of ether to resist crowding.* When ether is crowded it will attempt to flow out and to find room by force in surrounding space. In doing this it may have to crowd and dislodge other ether in its vicinity. When it thus disturbs this ether and crowds it, if there is not room for the escape of this crowded ether without resistance and the temperature is raised, but if the crowded ether of the atom's-sphere can escape readily we call the process a lowering of the temperature, or cooling.

Cooling therefore means unresisted outflow of Ether.

Ether existing all round us in a normal state may be called *free ether.* Ether enclosed by force in limited space surrounded by material atoms is imprisoned or stored ether: its greater or less degree of crowding or storing means degrees of heat, and the quantity of crowding among the atoms indicates the specific heat of these atoms, and sometimes the specific heat of that kind of matter.

Thus heat is not a thing or substance, it is *stored or crowded ether.* Heating is the act of forcing or confining ether in a given space, and cooling is only the escape or flow of ether out of crowded into free space.

What power have atoms to crowd or force Ether.

Just as a brick may build up a wall and form an enclosure for a reservoir, storehouse or prison, so may groups of atoms be arranged round a centre enclosing and confining imprisoned ether; but in the history of the universe it appears that all the storing of ether within the prison cells of matter has been done long ago, and that all the phenomena we now witness in nature are only the opening up of old storehouses whence the ether is continuously flowing in a stream of warmth as the essential condition of our life. What it is useful to discriminate is the use which we may make of this stored force to accomplish what we want. We can lay hold of some stores of ether and empty them out. We do that when we burn fuel, for burning is merely the breaking up of atomic stores of ether, the tearing of them asunder; and what we call work is only the application of ingenious modes for utilising the escaping ether to do this work. The ether once escaped to free space never returns to work again.

Fire.

One of the highest inventions of man, and one of the greatest of God's gifts to man, is fire. I take some atoms of steel and some crystals of flint, I crush them upon each other and tear open some cells, and out rushes the imprisoned ether. I have previously placed together bits of wood and blocks of coal: the released ether from the flint and steel rushes in among the wooden cells, and finding them also filled with imprisoned ether tears them open, their inmates acting in like manner with the ether in the coal. This imprisoned ether violently released spreads everywhere its intense force, and this intense flow out of ether is heat, but the attendant phenomena of heat we have yet to consider.

Heat is not flame nor light. We can have heat without light, and light without heat. Heat and light, fire and flame, have much to do with one another, but are not one and the same. I have defined heat as ethereal force imprisoned or flowing out of prison. Degrees of heat measure the speed or intensity of the rush out. I would define light as the waves of the ocean of ether, produced by the shock of the escaping ether, and spreading wide abroad through that ocean. It is the shock or pulse given to the ether by a convulsion as distinguished from the slower, narrower flow outwards of the ether which is heat. The heat is local and comparatively slow. The light is widespread and quick.

Any sudden outburst of imprisoned ether does two things; it flows out and fills its neighbourhood, forcing and pushing away the occupants of the space nearest to itself. That is its local effect on surrounding matter. But quite a different effect is produced on the ocean of ether beyond. The bursting of each prison cell sends out into this ocean a shock, beat or pulse like that which the heart sends into the arterial blood. This pulse gives to the larger expanse of ether, next and before it, the impulse which it received, and so shock after shock, and pulse after pulse, is propagated through the ether, and forms what we call light. Ether streams are heat, ether waves are light. The difference between the stream and the wave, or heat and light, are the same as the difference between the Gulf Stream and the ocean tide. In the one case the fluid moves and is removed far away from one place, and left at rest at a distance. In the other case the fluid stays on its own side of the world, but the *force* travels afar (leaving the fluid behind) and transfers itself by wave motion through the ocean to the other side of the globe, delivering its *force* on the shore.

In like manner as the stream of ether flowing out of its prison deluges with heat the vicinity of a fire, so does the *shock*, which strikes out in flame, send its violent impulse into the nearest ether, which it lights up with bright flame; and this shock, propagated through the ether with the speed of millions of miles an hour, carries to our eye the same clash which the ether received, giving to it the image of its source.

Capacity of Atom's-Spheres.

The use we make of nature's store of ether is to let it flow out of its reservoir into a place where it is wanted.

The place we wish to fill with ether may be an atom's-sphere of some specific kind of matters—oxygen, hydrogen, carbon, sulphur, zinc, iron, lead, gold, water. The atoms we mostly use in our application of ether are those of hydrogen and oxygen, in the shape of water. An atom of hydrogen weighing I. in its sphere II. is enclosed by force by the eight atoms of oxygen (all in one sphere), and the result is nine atoms of matter in a sphere of double the volume; and the question we now ask is, How much room is left for ether to occupy in the separate spheres before they are united, and in the united spheres afterwards? This question is called the question of specific heat of different kinds of matter, and sometimes its capacity for heat. We shall ask in each case how much room is left for ether within each atom's-spheres, beginning with an atom of hydrogen, and calling it sphere II. This sphere is double the size of an ordinary sphere; the size of the ordinary sphere of other atoms and the quantity of matter constituting the central atom is the smallest of all known atoms. Oxygen, although it occupies a sphere of half the size, has a group of eight atoms, each as heavy as the hydrogen atom. We now desire to know how much room is left vacant for ether in the hydrogen sphere and how much in the oxygen sphere?

Putting aside the size of the atoms, there is plainly twice as much room, and therefore twice the capacity for ether in the hydrogen sphere that there is in the oxygen spheres; and if we unite the double sphere and the single sphere into one, all full of ether, we should have a threefold sphere. But the power of the oxygen draws together this triple sphere, so as to reduce it to a twofold sphere, and we have nine atoms in a double atom's-sphere, or nine in II., and the question is now, what has become of the other ether sphere? Happily, experiment has given the answer—*one sphere of ether has been forcibly* turned out by the compressing power of the oxygen, and we can now measure the amount of ether contained in one atom's-sphere, and arrive at a very remarkable measure, which is—

That all atom's-spheres are of like bulk, or contain equal room.

That 64 *may be taken as a measure of that room or capacity to hold ether and atoms.*

We find it convenient to subdivide this space into hundred parts, so as to measure the contents of each atom's-sphere by the number

6400.

We now wish to know how much of an atom's-sphere is occupied by the bulk of the atom, and how much by the ether? Experiment shows that for every increase in a number of atoms there is a corresponding diminution of room for ether, and the proportion is so constant as to lead to the conclusion that all the atoms occupy like portions of the ether-sphere, and that the room left for the ether is exactly the reverse of that occupied by the atoms.

As example, take one oxygen sphere. It contains eight atoms, and these eight occupy eight of our sixty-four measures, leaving fifty-six out of the sixty-four unoccupied and free for the inflow of ether, and thus the eight atoms occupy eight units, and the ether fifty-six units.

Let us next take sulphur with its sixteen atoms in a group, which leave therefore only forty-eight units free for ether; each atom of sulphur thus leaves room for three volumes of ether, and each atom with its volume of ether is 16 times 4 = 64.

These examples tend towards the important conclusion that the capacities of material atoms and their spheres for containing ether are exactly in the reverse proportion of their *weights, masses, and attractive forces*, and that bodies of the greatest atomic weight have the least room for ether, and therefore the least capacity for heat.

This law of the inverse ratio of capacity for heat to atomic weight must exercise great influence on our mode of thought as to the constitution of matter. Are atoms of the different kinds of matters like or unlike? Are they the same, grouped in different numbers, set at different distances, arranged in different forms, and so giving rise to effects which are quite unlike? Or are the atoms of the sixty-four kinds of matter individual natures, each radically and irreconcilably distinct?

The law we have just been developing seems to favour the conclusion that the ultimate elements of matter may be like or identical, and that the sixty-four kinds of matter which we call elementary may really be only sixty-four different modes of numbering and grouping together atoms of like nature set in different combinations. This is quite a conceivable form of thinking out the unity of nature and the variety of matter, and therefore it will be wise to consider how out of like atomic units we could build up atomic groups unlike each other in form, bulk, and force.

We have seen how two like atoms placed at a distance attract each other more and more in a twofold proportion to their nearness, and we have seen how when they come to a certain distance they are stopped, and kept apart by an opposing force, which increases in the threefold degree of the nearness, and finally stops further approach.

This distance at which the opposing forces balance, is a primary element in ruling the results of our future grouping, and we may call it *the standard* or *normal distance* of *atoms*.

Let us conceive a multitude of atoms all alike, all possessing this force, all at the same distance from each other. That will be a kind of matter the nearest to hydrogen that we can conceive, and to conceive it most clearly we might think of a multitude of glass balls, each having in its centre a small black speck representing the atom, while the transparent ball would represent the sphere of ether. Further, conceive that these balls are not hard but soft and springy. Imagine a layer of these covering a table, a second, a third, any number of layers each on the other, and each layer pressed down and flattened by all the others resting upon it. That will fairly represent hydrogen.

Shall we now suppose every black atom at the centre of the sphere to be one indivisible round ball, or shall we fancy it made of two halves, male and female, positive and negative, or each in some manner fitted for that other, and only for that other? That is a question which arises irresistibly from many formulæ of chemistry. This question must be now postponed, but cannot be avoided.

We now ask how, out of single indivisible atoms we can build up new forms and natures?

New Forms and Natures of Atoms.

Take as the basis of our theory one elementary fact. Take 100 atoms of hydrogen and 800 atoms of oxygen. Mix them altogether in one box without order or arrangement. The hydrogen atoms, in their double spheres, are larger and lighter; the oxygen, smaller and heavier, so that the 800 will lie on the bottom and the 100 will swim on the top. A curious change will now take place. The 100 hydrogen atoms seem seized with a strong desire to occupy a larger space of the box, and to assist each other to take possession of it. For this purpose those lower down are pushed by those above them in between the atoms of oxygen, and then each of these hydrogen atoms pushes on and on, and behind, and in front, until at last the widespread atoms occupy the whole box, all standing at equal distances from each other. One consequence of this diffusion and dispersion of 100 atoms throughout one box is a certain regularity in the space within the box, so that each individual atom must occupy its own cell with about an equal space around it.

Meanwhile, what has become of the 800 atoms of oxygen which were lying below the thin layer of hydrogen? They were at the same time seized with the same desire as the hydrogen atoms to occupy the whole box. They pushed each other upwards row by row and layer by layer, in between the hydrogen atoms, thus in like manner diffusing themselves throughout the whole volume of the box, all at the same distance from each other, and each occupying an equal space.

We will now think out the nature of this orderly distribution. Conceive the box to measure a little more than nine inches every way (9·28). This box will contain 800 square cells, each an inch every way, therefore each oxygen atom would have an inch of room to itself, leaving no room unoccupied. But if the hydrogen be strong enough to take forcible possession of 100 places in between the atoms of oxygen, each group of oxygen occupying eight inches might, by very slight crowding, give up a portion of its cell so as to form one central cell for the hydrogen, and if each of the eight gave $\frac{1}{4}$ of its cell, the victorious intruder would get twice as much space as each oxygen atom held, and the whole space of the box would be as symmetrically occupied as a beehive.

We thus see how, by the enclosure of two different kinds of atoms in one limited space, they can, by their innate power of pushing (each their own kind), intrude into the space occupied and diffuse themselves symmetrically all through it. In this arrangement it is worth noticing that each hydrogen atom is twice as far from its own kind as each oxygen atom from its kind.

This arrangement is the result of the relation between the number two and the number eight. Two represents a double distance, while eight measures the space resulting from that double distance. A square box whose edge is twice the length of another holds eight times the quantity; therefore the atoms of hydrogen, having pushed each other asunder twice as far as the atoms of oxygen have done, have thereby left the eightfold space required.

All that we have said has reference to relative distances, and would be equally true if the atoms were $\frac{1}{1000}$ or $\frac{1}{1.000.000}$ of an inch apart, and were in the conditions of solid, fluid, and gaseous.

We are further led to the conclusion that each kind of atom must be endowed with some exact measure of repulsive force balancing the attractive force within it, and it is to these measures of force that we must look for the cause of the arrangement of the atoms in regular forms and in settled numbers.

Free Forms of Atom Groups.

We will next inquire what are the free forms that atoms would take of themselves when uncontrolled by limit of space. We will again conceive our eight equal atoms of oxygen, each the centre of eight equal corners, set together as a closed box, and the atoms equally parted, and then suppose that some strong external force squeezes these particles together into one central space in the midst of the eight. What will be the new relation of the eight atoms? What are their places, distance, and shape? First, they will not stay in the eight corners, but will take a triangular form, for this reason, that by attraction they will be nearer together in the triangular than in the square form. Second, the smallest number of atoms that can hold a fixed form by a given force at fixed distance is four. Third, the stable form of four in fixed places at fixed distance is a

Four-faced,
Four-cornered,
Six-edge figure,

commonly called a tetrahedron.

The second stable form is that of eight atoms, forming a very dissimilar group, which has

Six faces,
Eight corners,
Twelve edges.

We have thus two separate independent forms in which eight atoms can be grouped,—the four-faced and the six-faced,—and these same atoms may also be grouped in two groups of four, or one group of eight, and in each case there is a separate angle left for the place of each atom.

The simplest shape which a group of four atoms could take could be made thus:—Conceive three atoms, each in its own sphere, to be laid close together on a flat board. The centres of the three spheres would all be at some distance from each other. That distance would be two radii of two spheres, or one diameter of one sphere, and three diameters meeting at three corners make a perfect triangle, which we will call the base of our group. Let us now take our fourth atom in its sphere, and set it on the top of the united group. It will exactly fit into a hollow, left on the upper side of the three spheres, and will be set fast in that place, exactly at equal distance from the centres of the three spheres. Thus there is formed a permanent stable group of four atoms, in four spheres, at fixed distance on all four sides alike. This will be the exact relation of all the particles of any united mass of pure unmixed atoms left free.

This symmetrical arrangement of atoms represent the simplest state of aerial matter that we can conceive, for all the atoms are in like spheres, at like distances, in perfect balance of force, and regular line and order.

The bond of number is still wanting. To introduce grouping by number we must introduce some *cause.* This cause must distinguish some atoms from others, and this distinction must have the effect of drawing some together, and by doing so, parting some from others. Some power or force must select the first four atoms and make a beginning, and drawing these four nearer together, pushing them into one group, squeezing their four spheres into one, and so creating one central atom out of four others, and one large sphere out of eight smaller ones. Thus we have got the smallest group of atoms which can exist and make up one special substance growing out of the number four.

The consequences of this first act are immediate and manifold. The four atoms drawn together were thereby drawn away from their nearest neighbours all round. These neighbours cut off from their connections on one side were immediately drawn towards their neighbours on the other side, and thus each solitary atom rushed into the nearest place between the next three, and in this way each bereaved atom entered into close bond with three of a new family of four. Thus we have several groups of united atoms, each in their united sphere formed all round the first one.

This process cannot end when once begun. The union of these new groups has cut off all union with their outside neighbours, and each isolated one of these has rushed into the arms of his neighbour, and series after series of dislocation and bonds are successively wrought through the whole mass, until at last all are included in their groups of four with a fourfold atmosphere, enclosing each group, and what were formerly undistinguished atoms, all alike, are now families of the number four, or as groups of ozone, or any other chemical name possessing the qualities of the atomic groups just described, or, as it is termed, the "specific numbers of each substance."

It is important to notice that each of the four atoms in the one united sphere is no longer in the centre of the sphere, but that each is part of a group set in order round the centre of the joint atmosphere, in which all are now enclosed. It is thus plain that a radical change has been effected, for while the four atoms are held together by a closer bond holding them nearer than before, their larger sphere keeps off their outside neighbours to a wider distance, therefore there remains no chance of their ever regaining their original nearness to one another now that they have once been symmetrically set apart, except by the intervention of some new cause.

A like but different process to what we have seen, followed with the number 4, may be followed by the number 8. Conceive two groups of four to be powerfully pushed together by some external cause with such force that the two groups are finally compelled, in spite of their mutual repulsion, to become interlocked and to form a new permanent stable group of eight. This group of eight has the remarkable quality that each atom is at the same distance from three powerful neighbours, and that it holds each of the three firm and fast in its place, just as they hold him firm and fast in his place. That this should happen simultaneously to each and all of the eight is a remarkable quality belonging only to this one number and to this special form.

The form we have now got has eight corners, each occupied by one atom. From each corner runs three edges or lines, forming the ways from each atom to its three next neighbours ; and three such lines form the edges of six three-cornered faces, enclosing a space between the centres of the eight atoms, making this a flat-faced enclosure, having eight corners, twelve edges, and six faces.

This enclosure has a marvellous quality, radically different from and even contrary to the nature of the form of the number four. Our primary form of four was stable in shape and unchangeable in bulk. The new form of eight, with all its other qualities unchanged, may be changed into any other bulk without altering the size of any of its sides, and maintains the equidistance of each atom from three others. This variability of bulk with permanence of dimension, confers upon this number of eight its power of forming a great number of chemical combinations. The result is to give nearly omnipotence to oxygen, enabling it to manufacture out of other substances multitudinous, dissimilar combinations.

We have still two duties to perform—first, to analyse the nature of this eight-cornered symmetric arrangement of enclosing atoms and of the space within them ; and second, to consider the relation of its external form to the far larger sphere which surrounds it. This last may be as large as one, two, or three thousand times the bulk of the space enclosed within the atoms ; but it might also be reduced, and probably often is reduced, to seven or eight times the interior bulk. In the one case it may form a dense body, like diamond or gold; while in the other state it may be like steam or air.

These two problems are in fact the

Geometry of the Atomic Enclosure
and the
Geometry of the Atomic Sphere.

Geometry of Atomic Enclosure.

There are two radical forms the atomic enclosure can take without any deviation from the standard atomic distance; one we may call the square form, the other the sharp form.

The square form is that of a box or cube. In each of the eight corners we have set one atom; each atom has three lines going out from it, and at the end of these lines are three other atoms, all at the same distance from the first. We can speak of this atom and its three neighbours as one group; and if we carefully notice this group we shall see that it forms a cell or chamber of four sides or faces, four corners and six edges, and would if removed form a cell by itself.

Now this form admits of being divided into three cells of precisely equal bulk by parting lines all going from one corner to each of the other corners, and the three divisions thus made can each be subdivided into two by a diagonal line. The lines thus drawn give to each of the six cells four faces, four corners, and six edges. They are thus of the same nature as the original cell of four, but of a totally different shape.

Let us now suppose that a new atomic substance has been found ready to take possession of the same space, which is held by the eight atoms, and let us suppose that this new substance is formed of quite different numbers, six being its characteristic.

We have here six chambers to let and six tenants for them, and since the owners of the house occupy only eight corners, the six incoming tenants may be centered in the six vacant chambers without inconvenience. Thus the cubic form is admirably fitted to accommodate eight atoms of oxygen, and to leave six other places symmetrically arranged for six atoms of carbon, each occupying the centre of its chamber. This is an arrangement by which one of the most powerful combinations in chemistry is formed.

We will now describe the sharp form of the *No. 8 enclosure.* The sharp faces of this form are made of two perfect equilateral triangles; each forming together a lozenge, and six of these lozenge faces form the enclosure. Thus under a very different appearance, we have still a perfect symmetric form having six faces, eight corners, and twelve edges, all alike in size, but in shape radically different, the difference consisting mainly in the fact that all the faces of the former were right angled, and that all the new faces have their sharp angles measuring two-thirds of a right angle, and their blunt angles measuring four-thirds of a right angle. The form is thus much elongated in one direction, which we may call its axis, and much narrowed in the cross direction.

In this change of form we arrive at a quality in which the square form was deficient. Two atoms at two sharp points have been carried far asunder to nearly double their former distance, and form the extremities of the axis or poles of the form, while the remaining six atoms form two groups of three, brought much closer together right across the axis, and while each of the three maintains its distance unchanged from the nearest pole, its distance from its neighbour is diminished by one-seventh part, so that whereas formerly each atom was only at the standard distance from three other atoms, each one of the central atoms is now at an equal distance from six other atoms. It is this change in structure which gives to the sharp form the stability and polarisation which enables its atoms to take and to keep sharp bright crystalline forms, and also enables them to exert the strength of solid bodies which we call rigidity. The changes we are describing are those which take place in the atoms of water when they cease to flow and assume fixed place, distance, and direction, as in ice.

The next substance we have to instal in the chambers of the oxygen is that characterised by the number seven—namely, nitrogen. This installation is an easy process, for as we had already found room for one atom of hydrogen in the centre, with eight atoms of oxygen around it, let us place the seventh atom of nitrogen in the same central place. The other six atoms can take the six places formerly occupied by the six atoms of carbon.

Thus the combination of oxygen and hydrogen, of oxygen with carbon, and of oxygen and nitrogen, have been provided by the numerical arrangement of atoms; and as nearly the whole of the vegetable and animal kingdoms are made up of those four substances in combinations of the numbers 1, 8, 6, 7, or in their doubles, 2, 16, 12, 14, or in multiples of these, we can make all the multitudinous combinations of nature by lodging 2, 3, or more individual atoms in each of the cells which we have heretofore allotted to one.

Geometry of the Atomic Sphere.

We now proceed from the geometry of the atomic enclosure to that of the sphere left around and outside it, and in doing so we must notice the difference between the aspect of this enclosure according as it is occupied by a greater or less volume of atomic elements. When the enclosure is crowded to its fullest, and distended by force, it is of the square form or cube, but when less full, or when the forces without compress it, it takes the polarised or sharp form already described; and when it takes the perfect sharp form, characterised by the equilateral triangle, the inner contents or bulk of the enclosure are smaller than those of the square form in the proportion of 65 to 100, or almost exactly two-thirds of the bulk.

This is an important observation, because it so happens that the one atom of hydrogen required to form water (or in the first place steam) before it enters into the centre of the oxygen cell, is twice as large as it afterwards becomes under the enclosing force of the oxygen, so that there are two bulks of hydrogen and one of oxygen, or three bulks enclosed at starting, though the three bulks are afterwards compressed into two.

Now this compression is exactly what happens when the square enclosure is transformed into the sharp enclosure, as when the cube of steam becomes the crystal of ice.

Let us now turn our attention to the outer sphere. This outer sphere may be 10 or 1000 times the bulk of the atomic enclosure, and we desire to know what is happening there. Let us take as an illustration the substance called sulphur; it is characterised by the double of 8, 16, and therefore has an affinity for oxygen. Let us conceive the sixteen atoms of sulphur to be grouped in four; they would form the same shape as the elementary form of four corners, four faces, six angles. Each of these four groups of sulphur, having their faces of the same form as the *sharp* oxygen enclosure, would, if set upon four of its outside faces, exactly fit, and thus we might have, without disturbance, a triple combination of sixteen atoms of sulphur on the outside of the oxygen, to which we might add one atom of hydrogen in the inside, or six of carbon, or seven of nitrogen, or, as there are six outside faces of the oxygen, we might take six groups of sulphur, with four atoms in each, and thus combine twenty-four atoms of sulphur.

We may mark them as follows :—

Outside.	Enclosure.	Within.
4 × 4 Sulphur.	8 Oxygen.	1 Hydrogen.
4 × 4 „	8 „	6 Carbon.
4 × 4 „	8 „	7 Nitrogen.
	Or,	
4 × 6 Sulphur.	8 Oxygen.	1 Hydrogen.
4 × 6 „	8 „	6 Carbon.
4 × 6 „	8 „	7 Nitrogen.

The Wave of Translation in Ether.

There is radical difference between our relation to a wave in water or metal and to a wave in ether or air, which is, that we are on the outside of the two first and in the inside of the other two. In the former case the wave phenomenon is a visible external motion; in the latter case we must seek our information from new sources.

We must imagine ourselves placed in the bottom of the sea while a solitary wave of water is passing us. We might feel it by being lifted out of our place and set down in a new place further forward. We might observe it if we had a neighbour by being pushed closer upon him and then released. We might also observe it by being first lifted above our place, and then set down in it again. The process of a wave motion, therefore, to us in it, would be change of place forward, removal upward, and return downward, compression and release, approach and separation.

Let us now examine the effects which might be produced on matters around us by this same wave. Bodies around of like bulk and weight with ourselves, might undergo like changes and continue in the same relation to us, but those same waves might utterly change our relations provided the natures of the bodies were different from ours. A body of less bulk and more weight would be moved through a less distance than one of less weight or more bulk; and it might be, that a series of such waves would produce a powerfully separating and sorting effect. If a heap of various sorts of matters were subject to the transmission of a succession of solitary waves, the mixed heap would be found removed and rearranged, one sort being carried and left farthest, a second sort next, and the last sort behind. This we may call analysis by wave motion.

Another effect by wave-motion might be a piling together or building up of matters one above the other. In the process of being lifted up and let down, an accumulation in one place might obstruct the forward motion, and so by successive lifts matter might be piled up higher and higher, but built up in lines parallel to the wave-crest.

With other matters of a different nature some might be compressed and crushed, others in compression might be broken, while others enclosing elastic matter might be burst asunder. Thus the solitary wave might become the engine of rearrangement, building up, separation, or destruction.

We have next to consider certain changes which the wave itself may undergo from contact with adjacent and surrounding matter. If we suppose the bottom of the sea of water or of ether to be smooth and level, the wave travelling along such a bottom would travel without change ; but if the bottom were to shelve upwards or downwards, so as to shallow or deepen the water, the wave might go slower or faster, and the water-motion might be affected quite differently from the wave-motion, as in a slower wave the particles of water might be driven faster, and in a quicker wave slower. In water shallowing upwards the wave travels slower, the wave crest rises higher, and the movement of the particles on the bottom grows more rapid, while in the deeper water the wave grows more rapid, but the water-motion slower and gentler.

It follows, therefore, that the effects produced by a solitary wave on under-water matters may be radically altered by shelving of the bottom, and it is equally important to know that by stopping the sides of a channel a wave may be so doubled upon itself as to have a twofold, fourfold, or eightfold force, thereby gaining an increased height, and delivering its force with the increased velocity due to that height. Thus, by giving proper inclosures to a wave-channel, its power of affecting other matter may be greatly increased.

Further changes may be produced upon a wave by the well-known process of reflection, that is to say, that a resistant obstacle, having a flat place set fast in front of a wave, will send it back in a new direction, without altering its character. There will only be a complete reversal of the elements of its nature.

But instead of complete, there might be partial reversal. If in front of a wave a series of upright piles were set, each portion of the wave which struck the pile would be turned back; all the portion which passed between the piles would rejoin and form a new wave, going in the opposite direction. This process would apply equally to a grating; and if we conceive the portions of the grating and its openings to be equal, then a wave will be sent backwards exactly equal to that sent forwards, and the analysis of a wave into two opposites will be achieved.

Thus we see how by the surroundings of the solitary wave its forces may be modified in direction and in speed, and these modifications will enable us to make corresponding changes in the effects we may require it to produce in other matters.

Thus to conceive the effect of an ether wave upon matter, we must ascertain the condition of the ether and the condition of matter, and of differing matters in relation to each other, before proceeding to absolute conclusions as to effects produced by an ethereal wave upon material atoms, differing as they do in nature, in weight, and in volume.

The Solitary Wave in the Ocean of Ether.

The main conditions which affect the propagation of the wave are the attraction and repulsion forces due to the nature of each element, and the bulk and form of the mass composing the element. In a metal, whether it be solid or liquid, the attractive and repulsive forces are both very great. In water and like liquids the repulsive force remains as great, while the attractive force is reduced to $\frac{1}{10}$ or $\frac{1}{20}$. When we go to air the attractive force is diminished to $\frac{1}{1000}$ part that of water, and to $\frac{10}{1000}$ part that of metal, while the repulsive force remains undiminished; and when at last we arrive at ether, no attractive force seems to remain, or if it has any in it, it is only $\frac{1}{100.000}$ part of $\frac{1}{1.000.000}$ of the attraction of air. While the repulsive force not only remains undiminished, but shows itself far more highly developed than in any other kind of matter.

But although the ether may appear, when removed from all other matter, to be free from all bond of attraction as between the particles of its own substance, it is still conceivable, and even probable, that the atoms of other matter may have a strong attraction for it, and that may imply the existence of a reciprocal force in the ether, though an infinitesimal one, so small as in all ordinary cases to be neglected.

That the ether should possess in some measure, though infinitely small an attractive force, binding ether to ether and ether to matter, is a supposition necessary in order to escape from another supposition far less probable, although it is necessarily one of two alternatives. When we conceive an ethereal element pervading all space around our sun and beyond our system, and when we conceive that ether to be endowed only with repulsive force, and to be utterly insensible to the attraction of our planetary system and of its stellar surroundings, we are led on to the conclusion that this repulsive force acting alone would only drive the ether onward and outward until at last it would be utterly dispersed and disappear. To prevent this, some philosophers have invented imaginary prison walls, built up all around the universe, thus shutting up, along with the imprisoned stars, a great storehouse of ether which would otherwise escape. The attractive force of our planetary system, if imparted in the minutest degree to the ether, would as effectually confine it in the form of an aerial sphere all around the stellar system, as our little globe confines our aerial atmosphere, in a far smaller sphere, than that round which our moon revolves. Ether, therefore, with predominant repulsion, need not be altogether deprived of that minute attraction towards the elements of the universe which may prevent its disappearance into endless void space.

We may therefore proceed to consider what the phenomena would be of an immense but not infinite ethereal ocean all around the planetary system, endowed with a penetrative force, entering into every portion of space, left free and subject to the attraction of every kind of matter. If this were so, our universe would not be a vacuum, but a plenum, and it would be possible to conceive how, through this plenum of ether, the most distant portions of the universe might be placed in direct communication with one another.

A great solitary wave once created in this plenum of ether would be able to carry communications in exact time and in given speed all through the ethereal atmosphere. These ethereal wave messengers would deliver different kinds of messages, according to the nature of the source in which they originated, and these messages, when delivered at a distance, would produce different effects according to the nature of the receptacles and the mechanism which were prepared to receive them. The waves sent out might be larger or smaller, they might be differently timed, following closer or wider at given distance. Thus they would carry with them a power of communicating the same impressions in ether waves, which the air waves carry in musical sounds, and these successive accurately timed ether waves, when they reached the place of their delivery, would create, in one kind of mechanism, one manner of motion, and in each other mechanism a different manner of motion. And just as we call the rhythm of air waves, music, so we might call the impulse of ether waves, electricity and the flow of stored ether, heat; all being the work of the same waves.

Thus we see that an ethereal atmosphere filling our universe would, by the simple means of the solitary wave of translation, receive from its distant portions (where forces exist with power of wave genesis) regularly timed waves with accurate intervals, which might give out work, store up force-matter, or communicate thought by symmetric time.

Velocity of the Wave of Ether transmitting to any distance the force given into it.

What would be the velocity of a great wave of force travelling through the ether?

At the risk of being tedious, I must here repeat my former statements, and draw a comparison between three equivalent oceans, one composed of a heavy metal like mercury, a second of lighter liquid like water, and the third of the aerial fluid of our atmosphere.

Our atmosphere is five miles high, regarded as of uniform density. An equivalent mass of mercury would form a mass $2\frac{1}{2}$ feet deep, while an equal mass of water would form one 32 feet deep. Thus we have three seas all of one weight, and we have next to create a large solitary wave of the first order, in each of these seas, and ascertain the differing velocity in each. In the lake of mercury if the wave were created by a quick, strong push, it would run uniformly forward at nearly six miles an hour.

The force given to the push in the water may be lighter, and the wave will travel at a uniform rate of twenty miles an hour.

To make a wave of similar force in air we must take a large cannon ball which when fired will dis-

APPENDIX.

REPORT ON WAVES.

By J. SCOTT RUSSELL, M.A., F.R.S.

Made to the Meetings of the British Association in 1842 and 1843.

Members of the Committee { SIR JOHN ROBINSON,* Sec., R.S., Edinburgh. J. SCOTT RUSSELL, F.R.S.

A PROVISIONAL report on this subject was presented to the Meeting held at Liverpool in 1838, and is printed in the Sixth Volume of the Transactions. That report was a partial one. It states that "the extent and multifarious nature of the subjects of inquiry have rendered it impossible to terminate the examination of all of them in so short a time; but it is their duty to report the progress which they have made, and the partial results they have already obtained, leaving to the reports of future years such portions of the inquiries as they have not yet undertaken."

The first of these subjects of inquiry is stated to have been "to determine the varieties, phenomena, and laws of waves, and the conditions which affect their genesis and propagation."

It is this branch of the duty of the committee which forms the subject of the present report. Ever since the date of that report, it has happened that the author of this has been so fully preoccupied by inevitable duty, that it was not in his power to indulge much in the pleasures of scientific inquiry; and as the

* I cannot allow these pages to leave my hands without expressing my deep regret that the death of Sir John Robinson has suddenly deprived the Association of a zealous and distinguished office-bearer, and myself of a kind friend. In all these researches the responsible duties were mine, and I alone am accountable for them; but in forwarding the objects of the investigation I always found him a valuable counsellor and a respected and cordial co-operator.

active part of the investigation necessarily devolved upon him, it was not practicable to continue the series of researches on the ample and systematic scale originally designed so soon as he had anticipated, so that the former report has necessarily been left in a fragmentary state till now.

But I have never ceased to avail myself of such opportunities as I could contrive to apply to the futherance of this interesting investigation. I have now fully discussed the experiments which the former report only registered. I have repeated the former experiments where their value seemed doubtful, I have supplemented them in those places where examples were wanting. I have extended them to higher ranges, and where necessary to a much larger scale. In so far as the experiments have been repeated and more fully discussed, they have tended to confirm the conclusions given in the former report, as well as to extend their application.

The results here alluded to are those which concern especially the velocity and characteristic properties of the solitary wave, that class of wave which the writer has called the great wave of translation, and which he regards as the primary wave of the first order. The former experiments related chiefly to the mode of genesis, and velocity of propagation of this wave. They led to this expression for the velocity in all circumstances,

$$v = \sqrt{g\,(h + k)},$$

k being the height of the crest of the wave above the plane of repose of the fluid, h the depth throughout the fluid in repose, and g the measure of gravity. Later discussions of the experiments not only confirm this result, but are themselves established by such further experiments as have been recently instituted, so that this formerly obtained velocity may now be regarded as the phenomenon characteristic of the wave of the first order.

The former series of experiments also contained several points of research not published in the former report, because not sufficiently extended to be of the desired value. Among these were a series of observations on the actual motion of translation of particles of the fluid during wave transmission ; these have since been completed and extended, and the results of the whole are now given.

The former report was inevitably a fragment. I have endeavoured to give to the present report a somewhat greater degree of completeness. For this purpose I have now incorporated under one general form all those results of the present as well as of all my former researches, which could contribute to the unity and completeness of the view of a subject so interesting and important. I have re-discussed my former experiments, combined them with the more recent observations, and thus, from a wider basis of induction, obtained results of greater generality. Until the date of these observations, there had been confounded together in an indefinite notion of waves and wave motion, phenomena essentially different,—different in their genesis, laws of propagation, and other characteristics. I have endeavoured, by a rigid course of examination, to distinguish these different classes of phenomena from each other. I have determined certain tests, by which these confused phenomena have been made to divide themselves into certain classes, distinguished by certain great characteristics. Contradictions and anomalies have in this process gradually disappeared; and I now find that all the waves which I have observed may be distinguished into four great orders, and that the waves of each order differ essentially from each other in the circumstances of their origin, are transmitted by different forces, exist in different conditions, and are governed by different laws. It is now therefore easy to understand how much has been hitherto added to the difficulty of this difficult subject, by confounding together phenomena so different. The characteristics, phenomena, and laws of these great orders I have attempted in the present report to determine and define.

The knowledge I have thus endeavoured to obtain and herein to set forth concerning these beautiful and interesting wave phenomena, is designed to form a contribution to the advancement of hydrodynamics, a branch of physical science hitherto much in arrear. But besides this their immediate design, these investigations of wave motion are fertile in important applications, not only to illustrate and extend other departments of science, but to subserve the purposes and uses of the practical arts. I have ascertained that what I have called the great wave of translation, my wave of the first order, furnishes a type of that great oceanic wave which twice a day brings to our shores the waters of the tide. This type enabled us to understand and explain by analogy many of the phenomena of fluvial and littoral *tides*, formerly anomalous (see Proceedings R. S. Ed., 1838); and thus do these wave researches contribute to the advancement of the theory of the tides, a branch of physical astronomy long stationary, but which has recently made rapid strides towards the same high perfection which other branches of predictive astronomy have long enjoyed, a perfection which we owe chiefly to Sir John W. Lubbock, to Mr. Whewell, and the co-operation of the British Association. It is the wave of the first order enumerated in this report which furnishes to us the model of a terrestrial mechanism, by means of which the forces primarily imparted by the sun and moon are taken up and employed in the transport of tidal waters to distant shores (see previous Reports of Brit. Ass.), and their distribution in remote seas and rivers, which they continue in succession to agitate long after the forces employed in the genesis of the wave have ceased to exist (see Report on Tides). This application of the phenomena of waves to explain the tides is not their only application to the advancement of other branches of science. The phenomena of *resistance of fluids* I have found to be intimately connected with those waves (see Phil. Trans. Edin. 1837). The resistance which the water in a channel opposes to the passage of a floating body along that channel depends materially on the nature of the great wave of the first order, which the floating body generates by the force which propels it, and its motion is materially affected by the genesis of waves also of the second order arising from the same cause. These waves are therefore important elements in the resistance of fluids, and acquaintance with their phenomena is essential to the sound determination and explanation of the motion of floating bodies. If to these two branches of science we add the useful arts, in which an accurate acquaintance with wave phenomena may be of practical value to the purposes of human life, we shall find that the improvement of *tidal rivers*, the construction of *public works* exposed to the action of waves and of tides, and the *formation of ships* (see Reports of Brit. Ass. *passim*), are among the most direct and necessary applications of this knowledge, which is indeed essential to the just understanding of the best methods of opposing the violence of waves, and converting their motion to our own uses. By a careful study of the laws and phenomena of waves, we are enabled to convert these dangerous enemies into powerful slaves. By such applications of our wave researches, we therefore extend our knowledge in conformity with the maxims of the illustrious founder of our inductive philosophy, who enjoins that we always study to combine with our *experimenta lucifera* such *experimenta fructifera*, that while science is advanced society may be advantaged.

The Nature of Waves and their Variety.

When the surface of water is agitated by a storm, it is difficult to recognise in its tumultuous tossings any semblance of order, law, or definite form, which the mind can embrace so as adequately to conceive and understand. Yet in all the madness of the wildest sea the careful observer may find some traces of method; amid the chaos of water he will observe some moving forms which he can group or individualise; he may distinguish some which are round and long, others that are high and sharp; he may observe those that are high gradually becoming acuminate and breaking with a foaming crest, and may notice that the motion of those which are small is short and quick, while the rising and falling of large elevations is long and slow. Some of the crests will advance with a great, others with a less velocity; and in all he will recognise a general form familiar to his mind as the form of the sea in agitation, and which at once distinguishes it from all other phenomena.

Just as the waters of a reservoir or lake when in perfect repose are characterised by a smooth and horizontal surface, so also does a condition of disturbance and agitation give to the surface of the fluid this form characteristic of that condition and which we may term the wave form. When any limited portion of the wave surface presents a defined figure or boundary, which appears to distinguish that portion of fluid visibly from the surrounding mass, our mind gives it individuality,—we call it *a wave.*

It is not easy to give a perfect definition of a wave, nor clearly to explain its nature so as to convey an accurate or sufficiently general conception of it. Persons who are placed for the first time on a stormy sea, have expressed to me their surprise to find that their ship, at one moment in the trough between two waves, with every appearance of instant destruction from the huge heap of waters rolling over it, was in the next moment riding in safety on the top of the billow. They discover with wonder that the large waves which they see rushing along with a velocity of many miles an hour, do not carry the floating body along with them, but seem to pass under the bottom of the ship without injuring it, and indeed with scarcely a perceptible effect in carrying the vessel out of its course. In like manner the observer near the shore perceives that pieces of wood, or any floating bodies immersed in the water near its surface, and the water in their vicinity, are not carried towards the shore with the rapidity of the wave, but are left nearly in the same place after the wave has passed them as before. Nay, if the tide be ebbing, the waves may even be observed coming with considerable velocity towards the shore, while the body of water is actually receding, and any object floating in it is carried in the opposite direction to the waves, out to sea. Thus it is that we are impressed with the idea, that *the motion of a wave may be different from the motion of the water* in which it moves; that the water may move in one direction and the wave in another; that water may transmit a wave while itself may remain in the same place.

If, then, we have learned that a water wave *is not* what it seems, a heap of water moving along the surface of the sea with a velocity visible to the eye, it is natural to inquire what a wave really is; *what is wave-motion as distinct from water-motion?*

For the purpose of this inquiry let us take a simple example. I have a long narrow trough or channel of water, filled to the depth of my finger length. I place my hand in the water, and for a second of time push forward along the channel the water which my hand touches, and instantly cease from further motion. The immediate result is easily conceived; I have simply pushed forward the particles of water which I touched, out of their former place to another place further on in the channel, and they repose in their new place at rest as at first. Here is a final effect, and here my agency has ceased—not so the motion of the water; I pushed forward a given mass out of its place into another, but that other place was formerly occupied by a mass of water equal to that which I have forcibly intruded into its place; what, then, has become of the displaced occupant? it has been forced into the place of that immediately before it, and the occupant which it has dislodged is again pushed forward on the occupant of the next place, and thus in succession volume after volume continues to carry on a process of displacement which only ends with the termination of the channel, or with the exhaustion of the displacing force originally impressed by my hand, and communicated from one to another successive mass of the water. This process continues without the continuance of the original disturbing agency, and is prolonged often to great distances and through long periods of time. The continuation of this motion is therefore independent of the volition which caused it. It is a process carried on by the particles of water themselves obeying two forces, the original force of disturbance and the force of gravity. It is therefore a hydrodynamical phenomenon conformable to fixed law. I have now ceased to exercise any control over the phenomenon, but as I attentively watch the processes I have set a-going, I observe each successive portion of water in the act of being displaced by one moving mass of water, and in the act of displacing its successor. As the water particles crowd upon one another in the act of going out of their old places into the new, the crowd forms a temporary heap visible on the surface of the fluid, and as each successive mass is displacing its successor, there is always one such heap, and this heap travels apparently along the channel at that point where the process of displacement is going on, and although there may be only one crowd, yet it consists successively of always another and another set of migrating particles.

This *visible moving heap of crowding particles* is a true *wave*, the rapidity with which the displacement of one outgoing mass by that which takes its place, goes forward, determines the velocity with which the heap appears to move, and is called the *velocity of transmission* of the wave. The shape which the crowding of the particles gives to the surface of the water constitutes the *form of the wave*. The distance (in the direction of the transmission) along which the crowd extends, is called the *length or amplitude* of the wave. The number of particles which at any one time are out of their place, constitute the *volume of the wave;* the time which must elapse before particles can effect their translation from their old places to the new, may be termed the *period of the wave*. The *height of the wave* is to be reckoned from the highest point or crest to the surface of the fluid when in repose.

Such is the wave motion—very different is the *water motion*. Let us select from the crowd of water particles an individual and watch its behaviour during the migration. The progressive agitation first reaches it while still in perfect repose; the crowd behind it push it forward and new particles take its place. One particle is urged forward on that before it, and being still urged on from behind by the crowd still swelling and increasing, it is *raised* out of its place and *carried forward* with the velocity of the surrounding particles; it is urged still on until the particles which displaced it have made room for themselves behind it, and then the power diminishes. Having now in its turn pushed the particles before it along out of their place, and crowded them together on their antecedents, it is gradually left behind and finally *settles quietly down in its new place.* Thus, then, the *motion of migration of an individual particle* of water is very different from the *motion of transmission of the wave.*

The wave goes still forward along through the channel, but each individual water particle remains behind. The wave passes on with a continuous uninterrupted motion. The water particle is at rest, starts, rises, is accelerated, is slowly retarded, and finally stops still. The *range of the particle's motion* is short; its *translation* is interrupted and final. Its *vertical range* and *horizontal range are finite.* It describes an *orbit* or path during the *transit of the wave* over it, and remains for ever after at rest, unless when a second wave happens immediately to follow the first, when it will describe a second time *its path of translation,* passing through a series of new positions or *phases* during the *period of the wave.* The motion of the particle is not therefore like the apparent motion of the wave, either uniform or continuous. The motion of the water particles is a true motion of translation of matter from one place to another, with the velocity and range which the senses observe. But the wave motion is an ideal individuality attributed by the mind of the observer to a process of changes of relative position or of absolute place, which at no two instants belongs to the same particles in the same place. The water does not travel, the visible heap at no two successive instants is the same. It is the motion of particles which goes on, now at this place, now at that, having passed all the intermediate points. *It is the crowding motion alone which is transmitted.* This crowding motion transmitted along the water idealised and individualised is a true wave.

Wave propagation, therefore, consists in the transmission from one class of particles to another, of a motion differing in kind from the motion of transmission. *Wave motion* is therefore transcendental motion; motion in the second degree; the motion of motion—the transference of motion without the transference of the matter, of form without the substance, of force without the agent.

It is essential to the accurate conception and examination of waves, that this *distinction between the wave motion and the water motion* be clearly conceived. It has been well illustrated by the agitations of a crowd of people, and of a field of standing corn waving with the wind. If we stand on an eminence, we notice that each gust as it passes along the field bending and crowding the stalks, marks its course by the motion it gives to the grain, and the visible effect is like that of an agitated sea. The waving motion visibly travels across the whole length of the field, but the corn remains rooted to the ground; this illustration is as apt as old, being given to us in the Iliad, at the conclusion of the speech of Agamemnon, beginning *Ὦ φίλοι, ἥρωες Δαναοί.*

> *Ὣς φάτο·* . . . .
> *Κινήθη δ' ἀγορὴ, ὡς κύματα μακρὰ θαλάσσης*
> *Πόντου Ἰκαρίοιο, τὰ μέν τ' Εὖρός τε Νότος τε*
> *Ὤρορ', ἐπαΐξας πατρὸς Διὸς ἐκ νεφελάων.*
> *Ὡς δ' ὅτε κινήσει Ζέφυρος βαθὺ λήϊον, ἐλθὼν*
> *Λάβρος, ἐπαιγίζων, ἐπί τ' ἠμύει ἀσταχύεσσιν·*
> *Ὣς τῶν πᾶσ' ἀγορὴ κινήθη.*—Il. II. 144–149.

In the examination of the phenomena of waves, we have therefore two classes of elements for consideration, the elements of the wave motion and the elements of the water particle motion. We may first examine the *phenomena of a given wave-motion*, its range of transmission over the surface of the fluid, the velocity of that transmission, the form of the elevation, its amplitude, breadth, height, volume, period. We may next consider the *path which each water particle describes* during the wave transit; the *form* of that path, the *horizontal or vertical range* of the motion, the variation of the path with the depth, the relation of each *phase of the particle's orbit* to each portion of the corresponding wave length. By this examination I have found that there exist among waves groups of phenomena so different as to suggest their division into *distinct classes.* I find that the general form of waves is manifestly different, one kind of wave making its appearance in a form always wholly raised above the general surface of the fluid, and which we may call a *positive wave*, and so distinguish it from another form of wave which is wholly *negative*, or depressed below the plane of repose, while a third class are found to consist of both a negative and a positive portion. I find them propagated with extremely different velocities, and obeying different laws according as they belong to one or the other of these classes, the positive wave having *in a given depth* *of water a constant and invariable velocity*, while another class has a *velocity varying* according to other peculiarities, and *independent of the depth.* Some of them again are distinguished by always appearing alone as individual waves, and others as *companion phenomena* or *gregarious*, never appearing except in groups. In examining the paths of the water particles corresponding differences are observed. In some the water particles perform a *motion of translation* from one place to another, and effect a permanent and final change of place, while others merely change their place for an instant to resume it again; thus performing *oscillations* round their place of final repose. These waves may also be distinguished by the sources from which they arise, and the forces by which they are transmitted. One class of wave is a *motion of successive transference* of the whole fluid mass; a second, the *partial oscillation* of one part of it without affecting the remainder; a third, the propagation of an impulse by the *corpuscular forces* which determine the elasticity of the fluid mass; and a fourth, by the *capillary forces* uniting its molecules at the surface.

These classes, so various both in their origin, cause, and phenomena, have not hitherto been sufficiently distinguished, but have either been unknown, or have been confounded with each other under the vague conception and general designation of wave motions. The following table is given as a *first approximation towards a classification of the phenomena of wave motion.* It comprehends all the waves which I have investigated, and sufficiently distinguishes them from each other. I find that water waves may be distributed into *four orders.* The *wave of translation* is the wave of the FIRST ORDER, and consists in a motion of translation of the whole mass of the fluid from one place to another, to another in which it finally reposes; its aspect is, a *solitary* elevation or a solitary hollow or cavity, moving along the surface with a *uniform* velocity; and hence it presents two species, *positive* and *negative*, and each of these may be found in a condition of *free* motion, or affected in form and velocity by the continual interference of a *force* of the same nature with that from which its genesis was derived. The wave of the SECOND ORDER is partly positive and partly negative, *each height having a companion hollow*, and this is the commonest order of visible water wave, being similar to the usual *wind waves*, in which the surface of the water visibly *oscillates* above and below the level of repose; these waves appear in *groups;* in some cases, as in running water, they may be *standing* elevations and depressions, and in others *progressive* along the surface, and like the waves of the first order, may be altered in form and velocity by the presence of a disturbing force, so as to differ from their phenomena when in a state of perfect freedom. The THIRD ORDER are met with under such conditions as agitate the fluid only to a very minute depth, and are determined by the same forces which in hydrostatics produce the phenomena of *capillary attraction;* and the FOURTH ORDER is that wave insensible to sight, which conveys the disturbance produced by a sonorous body through a mass of the fluid, and which is at once an index and a result of the molecular forces which determine the elasticity of the fluid. This classification has been adopted throughout the following paper.

TABLE I.

System of Water Waves.

ORDERS.	FIRST.	SECOND.	THIRD.	FOURTH.
Designation	Wave of translation	Oscillating waves.	Capillary waves	Corpuscular wave.
Characters	Solitary............	Gregarious ...	Gregarious	Solitary.
Species...	Positive .... Negative	Stationary ... Progressive ...	Free. Forced.	
Varieties	Free .... Forced	Free Forced		
Instances	The wave of resistance The tide wave ... The aërial sound wave	Stream ripple Wind waves... Ocean swell.	Dentate waves Zephyral waves	Water-sound wave.

An observer of natural phenomena who will study the surface of a sea or large lake during the successive stages of an increasing wind, from a calm to a storm, will find in the whole motions of the surface of the fluid, appearances which illustrate the nature of the various classes of waves contained in Table 1., and which exhibit the laws to which these waves are subject. Let him begin his observations in a perfect calm, when the surface of the water is smooth and reflects like a mirror the images of surrounding objects. This appearance will not be affected by even a slight motion of the air, and a velocity of less than half a mile an hour (8½ in. per sec.) does not sensibly disturb the smoothness of the reflecting surface. A gentle zephyr flitting along the surface from point to point, may be observed to destroy the perfection of the mirror for a moment, and on departing, the surface remains polished as before; if the air have a velocity of about a mile an hour, the surface of the water becomes less capable of distinct reflection, and on observing it in such a condition, it is to be noticed that the diminution of this reflecting power is owing to the presence of those minute corrugations of the superficial film which form waves of the *third order*. These corrugations produce on the surface of the water an effect very similar to the effect of those panes of glass which we see corrugated for the purpose of destroying their transparency, and these corrugations at once prevent the eye from distinguishing forms at a considerable depth, and diminish the perfection of forms reflected in the water. To fly-fishers this appearance is well known as diminishing the facility with which the fish see their captors. This first stage of disturbance has this distinguishing circumstance, that the phenomena on the surface cease almost simultaneously with the intermission of the disturbing cause, so that a spot which is sheltered from the direct action of the wind remains smooth, the waves of the third order being incapable of travelling spontaneously to any considerable distance, except when under the continued action of the original disturbing force. This condition is the indication of present force, not of that which is past. While it remains it gives that deep blackness to the water which the sailor is accustomed to regard as an index of the presence of wind, and often as the forerunner of more.

The second condition of wave motion is to be observed when the velocity of the wind acting on the smooth water has increased to two miles an hour. Small waves then begin to rise uniformly over the whole surface of the water; these are waves of the second order, and cover the water with considerable regularity. Capillary waves disappear from the ridges of these waves, but are to be found sheltered in the hollows between them, and on the anterior slopes of these waves. The regularity of the distribution of these secondary waves over the surface is remarkable; they begin with about an inch of amplitude, and a couple of inches long; they enlarge as the velocity or duration of the wave increases; by and by conterminal waves unite; the ridges increase, and if the wind increase the waves become cusped, and are regular waves of the *second order*. They continue enlarging their dimensions, and the depth to which they produce the agitation increasing simultaneously with their magnitude, the surface becomes extensively covered with waves of nearly uniform magnitude.

How it is that waves of unequal magnitude should ever be produced may not seem at first sight very obvious, if all parts of the original surface continue equally exposed to an equal wind. But it is to be observed that it rarely occurs that the water is all equally exposed to equal winds. The configuration of the land is alone sufficient to cause local inequalities in the strength of the wind and partial variations of direction. By another cause are local inequalities rapidly produced and exaggerated. The configuration of the shores reflects the waves, some in one direction, some in another, and so deranges their uniformity. The transmission of reflected waves over such as are directly generated by the wind, produces new forms and inequalities, which, exposed to the wind, generate new modifications of its force, and of course, in their turn, give rise to further deviations from the primitive condition of the fluid. There are on the sea frequently three or four series of coexisting waves, each series having a different direction from the other, and the individual waves of each series remaining parallel to one another. Thus do the condition, origin, and relations of the waves which cover the surface of the sea after a considerable time become more complex than at their first genesis.

It is not until the waves of the sea encounter a shallow shelving coast, that they present any of the phenomena of the wave of the first order (Report of 1838). After breaking on the margin of the shoal, they continue to roll along in the shallow water towards the beach, and becoming transformed into waves of the first order, finally break on the shore.

But the great example of a wave of the *first order*, is that enormous wave of water which rolls along our shores, bringing the elevation of high tide twice a day to our coasts, our harbours, and inland rivers. This great compound wave of the first order is not the less real that its length is so great, that while one end touches Aberdeen, the other reaches to the mouth of the Thames and the coast of Holland. Though the magnitude of this wave renders it impossible for the human eye to take in its form and dimensions at one view, we are able, by stationing numerous observers along different parts of the coasts, to compare its dimensions and to trace its progress at different points, and so to represent its phenomena to the eye and the mind on a small scale, as to comprehend its form and nature as clearly as we do those of a mountain range, or extensive country which has been mapped on a sheet of paper by the combination together of trigonometrical processes, performed at different places by various observers, and finally brought together and protracted on one sheet of paper.

As this great wave of the first order is not comprehended by the eye on account of its magnitude, so there is a wave of the *fourth order* which equally escapes detection from that organ, on account of its minuteness. By an undulation propagated among the particles of water, so minute as to be altogether insensible to the eye, and only recognised by an organ appropriate to that purpose, there is conveyed from one place to another the wave of sound. This wave, though invisible from its minuteness, is nevertheless of a nature almost identical with the wave of the first order. In air the sound wave is indeed the wave of the first order. It is only in liquids, when the measure of pressure of the fluid mass is different from the measure of the intercorpuscular force, that the phenomena of the wave of the first order is different from those of the fourth, and that we have one measure for the velocity of the water wave, and another for that of the sound wave. In a gaseous fluid, on the contrary, the measure of the pressure of the mass is also the measure of the intercorpuscular force, and the sound wave becomes identical with the air wave, the fourth order with the first.

SECTION I.—WAVE OF THE FIRST ORDER.

The Wave of Translation.

Character	Solitary.
Species	{ Positive. Negative.
Varieties , ...	{ Free. Forced.
Instances	{ Wave of Resistance. Tidal Wave—Sound Wave.

I believe I shall best introduce this phenomenon by describing the circumstances of my own first acquaintance with it. I was observing the motion of a boat which was rapidly drawn along a narrow channel by a pair of horses, when the boat suddenly stopped—not so the mass of water in the channel which it had put in motion; it accumulated round the prow of the vessel in a state of violent agitation, then suddenly leaving it behind, rolled forward with great velocity, assuming the form of a large solitary elevation, a rounded, smooth, and well-defined heap of water, which continued its course along the channel apparently without change of form or diminution of speed. I followed it on horseback, and overtook it still rolling on at a rate of some eight or nine miles an hour, preserving its original figure some thirty feet long and a foot to a foot and a half in height. Its height gradually diminished, and after a chase of one or two miles I lost it in the windings of the channel. Such, in the month of August 1834, was my first chance interview with that singular and beautiful phenomenon which I have called the Wave of Translation, a name which it now very generally bears; which I have since found to be an important element in almost every case of fluid resistance, and ascertained to be the type of that great moving elevation of the sea, which, with the regularity of a planet, ascends our rivers and rolls along our shores.

To study minutely this phenomenon with a view to determine accurately its nature and laws, I have adopted other more convenient modes of producing it than that which I have just described, and have employed various methods of observation. A description of these will probably assist me in conveying just conceptions of the nature of this wave.

Genesis of the Wave of the First Order.—For producing waves of the first order on a small scale, I have found the following method sufficiently convenient. A long narrow channel or box a foot wide, eight or nine inches deep, and twenty or thirty feet long (Plate I. fig. 1), is filled with water to the height of say four inches. A flat board P (or plate of glass) is provided, which fits the inside of the channel so as to form a division across the channel where it is inserted.

Genesis by Impulsion or Force horizontally applied.—Let this plate be inserted vertically in the water close to the end A, and being held in the vertical position, be pressed forward slowly in the direction of X, care being taken that it is kept vertical and parallel to the end. The water now displaced by the plate P in its new position accumulates on the front of the plane forming a heap, which is kept there, being enclosed between the sides of the channel and the impelling plate. The amount thus heaped up is plainly the volume of water which has been removed by the advancing plane from the space left vacant behind it, and if the impulse increase, the elevation of displaced water will increase in the same quantity. When the water has reached the height P_3, let the velocity of impulsion be now gradually diminished as at P_4, until the plate is finally brought to rest as at P_5; the height of the water heaped on the front will diminish with the diminution of velocity as at P_4, and when brought to rest at P_5 it will be on the original level. The total height of the water does not, however, subside with the diminution of the impulsion, the crest W_4 retains the maximum height to which it had risen under the pressure of the plane at P_3, and moves horizontally forward; and the smaller elevation produced by the smaller pressure at P_4 down to P_5 moves forward after W_5. This elevation of the liquid, having a *crest*, or summit, or *ridge* in the centre of its length transverse to the side of the channel, continues to move along the channel in the direction of the original impulsion; from the crest there extends forward a curved surface, W*a*, forming the *face* of the *wave*, and a similar surface, W*w*, behind the crest, is distinguished as its *back*. It is convenient to designate *a* as the *origin*, *w* as the *end* of the wave; and to designate the interval between *a* and *w*, the length of the wave in the direction of its transmission, its *amplitude*.

The kind of motion required for generating this wave in the most perfect way, that is, for producing a wave of given magnitude without at the same time creating any disturbance of a different kind in the water—this kind of motion may be given by various mechanical contrivances, but I have found that the dexterity of manipulation which experience bestows is perfectly sufficient for ordinary experimental purposes.

Genesis by a Column of Fluid.—This is a method of genesis, of considerable value for various experimental purposes, especially useful when waves of no great magnitude are required, and also when it is desirable to measure accurately volumes or forces employed in wave genesis. The same glass plate may be conveniently employed in this case as in the last, only it will now be used in the capacity merely of a sluice, and be supported by two small vertical slips fixed to the sides of the channel, so as to keep it in the vertical position, but to admit of its being raised vertically upwards as at G, Pl. I. fig. 2. There is thus formed between the end of the channel G and the moveable plate P_7, a small generating reservoir GP_7. This is to be filled to any desired height with water, as from w to P_7, and the plate being drawn up, as at P_9, the water of the reservoir descends to w, the level of the water of the channel, and pushing forward and heaping up the adjacent fluid, raises a heap equal to the added volume on the surface of the water; and this elevation is in no respect sensibly different either in form or other phenomenon from that generated in the former method, provided the quantity of water added in the latter case be identical with the quantity of water displaced in the former case.

This method of genesis by fluid column affords a simple means of proving an elementary fact in this kind of wave motion. The fact is this, that while the volume of water in the wave is exactly equal to the volume of water added from the reservoir, it is by no means identical with it. I filled the reservoir with water tinged with a pink dye, which did not sensibly alter the specific gravity of the water. The column of water having descended as at K, and the wave having gone forward to W_9, the generating column remained stationary at K, thus indicating that the column of water had merely acted as a mechanical prime mover, to put in action the wave-propagating forces among the fluid, in the same way as had been formerly done by the power acting by the solid plate in the former case of genesis by impulsion. Thus is obtained a first indication that this wave exhibits *a transmission of force, not of fluid,* along the channel.

Genesis by Protrusion of a Solid.—The quantity of moving force required for the wave-genesis may be directly obtained by the descent of a solid weight. The solid at L (fig. 3) may be a box of wood or iron, containing such weights as are desired, and suspended in such a manner as to be readily detached from its support. Its under surface should be somewhat immerged. On touching the detaching spring, the weight descends, and the water it displaces produces a wave of equal volume. If the weight and volume of solid thus immersed be equal to those of the water in the reservoir in the former case, it is found that the waves generated by the two methods are alike. It is expedient that the breadth and shape of this solid generator should be such as to fit the channel, as this removes some sources of disturbance. The results which are produced by this application of moving power are also convenient for giving measures of the mechanical forces employed in wave-genesis.

This method is especially convenient for the genesis of waves of considerable magnitude. With this view I erected a pyramidal structure of wood, capable of raising weights of several hundred pounds, over a pulley by means of a crane, and contrived to allow them to descend at will. This apparatus was adequate for the generation of waves in a channel three feet wide and three feet deep; and the same construction may be extended to greater dimensions.

Transmission of Mechanical Power by the Wave.—By the last two methods of genesis there is to be obtained a just notion of the nature of the wave of the first order as a vehicle for the transfer of mechanical power. By the agency of this wave the mechanical power which is employed in wave-genesis at one end of the channel, passes along the channel in the wave itself, and is given out at the other end with only such loss as results from the friction of the fluid. At one end, as of the channel G, fig. 4, there is placed the water, which, falling through a given height, is to generate the wave. At the other end, X, is a similar reservoir and sluice, open to the channel. When the wave has been generated as at K, and has traversed the length of the channel, it enters the receptacle X, and assuming the form marked at L, the sluice being suddenly permitted to descend, the column of water will be enclosed in the receptacle, and its whole volume raised above the level of repose nearly as at the first. The power expended in wave-genesis, having been transferred along the whole channel, is thus once more stored up in the reservoir at the other extremity. A part of this power is, however, expended *in transitu* by friction of the particles and imperfect fluidity, &c. When the channel is large, the sides and bottom smooth, the transmission of force may be accomplished with high velocity, at the rate of many miles an hour, to a distance of several miles.

Re-genesis of Wave.—In the channel AX, we have found the wave transmitted from A to X, and there the power of genesis transferred to the fluid column now stored up in the reservoir X. If we now repeat from the receptacle X the same process of genesis originally performed at G, elevating the sluice and allowing the fluid column to descend, it will again generate a wave similar to the first, only transmitted back in the opposite direction. This re-generated and re-transmitted wave may be again found in the primary reservoir of genesis as at G, and the same power, after having been transmitted twice through the length of the channel, be restored as at first in that channel, with only the small diminution of power lost *in transitu*. The process of re-genesis may now be repeated, as at first, and so on during any number of successive transmissions and re-transmissions.

Reflexion of the Wave.—This process of restoring the force employed in wave-genesis, and of re-genesis of the wave, may take place without the intervention of the sluices. The wave, on reaching the end of the channel G at X_7, becomes accumulated in the form of the curve *w x*. We have therefore the power of genesis now stored up in this water column, *w* L *x*, above the level L, and in a state of rest. By means of a sluice we may detain it at that height for as long time as we please. But let us suppose we do not wish to detain it, but allow the water column to descend by gravity as at first, it generates the wave by again descending, and transmits it back towards G, as effectually as if the reservoir had been used, or as the genesis when first accomplished. By the same process of *laissez faire*, the power of genesis will be restored at G, a water column elevated, the fluid brought to rest and allowed again to descend, again to effect genesis of the wave, and again transmit the force along the channel through the particles of the wave. The wave is said to be reflected, and it is thus shown in reference to the wave of the first order, that the process called reflexion consists in a process of restoration of the power of genesis, and of re-genesis of the wave in an opposite direction. In this manner there is to be obtained an accurate view of the mechanical nature of the reflexion of the wave.

Measure of the Power of Wave-Genesis.—If we examine the process of wave-genesis as at K, fig. 2, we find that the change which has taken place after the wave-genesis and before, consists virtually in a different arrangement of the particles of a given volume of water. The given rectangular column of water AP_{10} occupies after genesis the equal space AK. This, without regard to the paths in which the particles have proceeded to their new places, this descent is the final result and integral effect of the development of the power of the generating column. Take away from these two equal volumes of fluid the volume $g\,p$, common to both, and the remaining volumes w P and $p\;k$ are equal, and a given volume of water has effectively descended from P G w into K $k\;p$, and g_1 and g_2 being the centres of gravity; the quantity of power developed is measured by the descent of the weight of water through a height $g_1\,g_2$, or through half the depth of the generating reservoir, and is of course capable of generating in any equal mass of fluid a velocity equal to that which is acquired by falling through a space equal to one-half the depth, reckoned from the top of the generating column to the bottom of the channel.

Imperfect Genesis of the Wave.—The wave may be said to have imperfect genesis, as far as the purposes of accurate experiment are concerned, when it is accompanied by other wave phenomena which interfere with it. The precautions necessary to perfect genesis appear to be these, that the volume of water should not widely differ from the volume of the wave it is proposed to generate, and that the height of the water should not greatly differ from that of the wave; and even these precautions are scarcely sufficient for the generation of a perfect solitary wave in a case where it is extremely high. The reason is obvious.

Residuary Positive Wave.—In a case of genesis where the precautions mentioned above are not observed, the following phenomenon is exhibited. If, as in the case fig. 6, the volume of the generating fluid considerably exceed (in consequence of the length of the generating reservoir) the length of the wave of a height equal to that of the fluid, the wave will assume its usual form W notwithstanding, and will pass forward with its usual volume and height; it will free itself from the redundant matter w by which it is accompanied, leaving it behind, and this residuary wave, w_2, will follow after it, only with a less velocity, so that although the two waves were at first united in the compound wave, they afterwards separate, as at W_2 w, and are more and more apart the further they travel.

Disintegration of large Wave Masses.—Thus also by increasing the length of the generating column, there may be generated any number of residuary waves, and it is a result of no little importance, to just conceptions of the nature of the wave of the first order, that it be not regarded as an arbitrary phenomenon deriving all its characters from the conditions in which it was at first generated, but that it is a phenomenon *sui generis*, assuming to itself that form and those dimensions under which alone it continues to exist as a wave. The existence of a moving heap of water of any arbitrary shape or magnitude is not sufficient to entitle it to the designation of a wave of the first order. If such a heap be by any means forced into existence, it will rapidly fall to pieces and become disintegrated and resolved into a series of different waves, which do not move forward in company with each other, but move on separately, each with a velocity of its own, and each of course continuing to depart from the other. Thus a large compound heap or wave becomes resolved into the principal and residuary waves by a species of spontaneous analysis.

Residuary Negative Waves.—There is a method of genesis the reverse of the last, which also produces residuary waves, but they are thus far the reverse of the last in form, as they have the appearance of *cavities* propagated along the surface of the still water in the channel, and they move *more slowly* than the positive wave: we may give them the appellation of residuary negative waves. When the elevation of the fluid in the reservoir is great in proportion to its breadth (reckoned as amplitude), the descending column of genesis communicates motion to a greater number of particles of water than its own, but with a less velocity; these go to form a wave which is larger in volume than the column of genesis, and therefore contribute to the volume of the wave some of the water which originally served to maintain the level of the fluid or surface of repose; this hollow is transferred like a hollow wave along the fluid, and there may exist several such waves, which I have called residuary negative waves. But these waves do not accompany the primary wave, nor have they the same velocity. See O, fig. 16.

It is of some importance to note, that these residuary phenomena of wave-genesis are *not companion phenomena* to the primary wave or positive wave of the first order. They will be separately considered at another time; meanwhile it is to be noted that these residuary phenomena accompany only the genesis of the wave, but do not attend the transmission, as they are rapidly left behind by the great primary solitary wave of the first order. Certain philosophers have fallen into error in their conceptions of these experiments by not sufficiently noting this distinction.

It is worth notice, also, that besides these, many other modes of genesis have been employed; solids elevated from the bottom of the channel, vessels drawn along the channel, &c.; wherever a considerable addition is made to the height and volume of the liquid at any given point in the channel, a wave of the first order is generated, differing in no way from the former, except in such particulars as are hereinafter noticed.

Motion of Transmission.—The crest of the wave is observed to move along a channel which does not vary in dimension, with a *velocity sensibly uniform*, so that the velocity with which it is transmitted may be determined by simply measuring a given distance along the channel, and observing the number of seconds which may elapse during the transit from one end of the line to the other. This interval of time is sensibly equal for any equal space measured along the path, and hence we determine that the velocity of the wave transmission is sensibly uniform.

Range of Wave Transmission.—The distance through which a wave of the first order will continue to propagate itself, is so great as to afford considerable facility for accurate observation of its velocity. For accurate observations it is convenient to allow the early part of the range to escape without observation; for this purpose, that the primary wave, which is to be the subject of observation, may disembarrass itself of such secondary phenomena as frequently accompany its genesis, when that genesis cannot be accurately accomplished. A small part of the range is sufficient for this purpose, and the remainder is perfectly adapted for purposes of accurate observation, as it continues to travel along its path long after the secondary waves have ceased to exist. The *longevity of the wave of the first order*, and the facility of observing it, may be judged of from the following experiments, made in 1835–1837.

Ex. 1. A wave of the first order, only 6 inches high at the crest, had traversed a distance of 500 feet, when it was first made the subject of observation. After being transmitted along a further distance of 700 feet, another observation was noted, and it was observed still to have a height of 5 inches, and to have travelled with a velocity of 7·55 miles an hour.

Ex. 2. A wave of the first order, originally 6 inches high, was transmitted through a distance of 3200 feet, with a mean velocity of 7·4 miles an hour, and at the end of this path still maintained a height of 2 inches.

Ex. 3. A wave 18 inches high, moving at the rate of 15 miles an hour, in a channel 15 feet deep, had still a height of 6 inches, having traversed the same space in 12 minutes.

Ex. 4. Among small experimental waves of the first order, in small channels, I have selected one, whose crest being 1·34 inch high, in a channel 5·10 inches deep, was transmitted through a range of 1360 feet, and still admitted of accurate observation.

These examples serve to convey an accurate idea of the longevity of a wave of the first order. And this longevity appears to increase with the depth and the breadth of the channel, and with the height of the wave crest.

Degradation of the Wave of the First Order.—In the progress of a wave of the first order, it is observed that its height diminishes with the length of its path; the velocity also diminishes with the diminution of height, though very slowly. This degradation of height is observed to go on more rapidly in proportion as the channel is narrow, shallow, or irregular, and rough on the sides, and is diminished according as the channel is made smooth and regular in its form, or deep and wide. It is to be attributed to the imperfect fluidity of the water in some degree, but also to the adhesion of water to the sides. The particles of fluid near the sides and bottom are retarded in their motions, and the transmission takes place more slowly among them. The wave passes on, leaving in these particles a small quantity of the motion it had communicated, and of its force and volume, and in consequence of this there exists along the whole channel, over which the wave has passed, a residual motion or continuous residual wave, very small in amount, but still appreciable by accurate means of observation. The volume of the wave is thus diffused over a large extent along its path, where finally it has deposited the whole of its volume, and so disappears. This degradation is therefore the means by which the motion of a wave in an indefinite channel is gradually and slowly terminated. In the history of a solitary wave of the first order, the progress of this degradation is to be observed from the examination of Table II., column B, which gives the height of the wave as observed at every 40 feet along its path. In the first 200 feet this diminution amounts to about $\frac{1}{4}$ of the height at the commencement. At the end of the second 200 feet, the height is diminished by $\frac{4}{9}$ of the height at the commencement of that space. During the third space of 200 feet the degradation produced is nearly $\frac{1}{2}$ of the height of the wave; this appears to be the most rapid degradation, and in the next space of 200 feet it is little more than $\frac{1}{3}$; in the next, less than the third of the height at the beginning of that space. These successive heights are given graphically in Plate II. fig. 7.

The Velocity of Transmission of the Wave of the First Order.—The history of a single wave has sufficed to show us that the velocity with which its crest is transmitted along the channel is nearly that which a heavy body will acquire falling freely through a height equal to half the depth of the fluid. This is a very simple and important character in the phenomena of this wave, by which, when the depth of the channel is known, we may at once predict approximately the velocity of the wave of translation. The following are approximate numbers deduced from this conclusion, and which I find it convenient to recollect.

In a channel whose depth is $2\frac{1}{2}$ inches, the velocity of the wave is $2\frac{1}{2}$ feet per second.

In a channel whose depth is 15 feet, the velocity of the wave is 15 miles an hour.

In a channel whose depth is 90 fathoms, the velocity of the wave is 90 miles an hour.

These numbers are, however, only first approximations, for it is to be observed in reference to wave, Table II., that the wave, when its height is considerable, moves with greater velocity than when it is small. These numbers become accurate, if in the depth, the height of the wave be included.

The Height of the Wave of the First Order, an element in its velocity.—The height of the wave appears to enter as an element in its velocity, and to cause it to deviate from the simple formula A. Thus the velocity of the wave only coincides with the velocity assigned in Table II. when the height of the wave is inconsiderable.

I have found that this deviation is to be reconciled, without at all destroying the simplicity of the formula, by a very simple means. In order to obtain perfect accuracy, we have only to reckon the effective depth for calculation, from the ridge or crest of the wave instead of from the level of the water at rest; and having thus added to the depth of the water in repose, the height of the wave crest above the plane of repose, if we take the velocity which a heavy body would acquire in falling through a space equal to half the depth of the fluid (reckoning from the ridge of the wave to the bottom of the channel), that number accurately represents the velocity of transmission of the wave of the first order.

We have, therefore, for the velocity of the wave of the first order,

approximately $v = \sqrt{gh}$, ... A
accurately $v = \sqrt{g(h+k)}$, ... B

where v is the velocity of transmission,
g is the force of gravity as measured by the velocity which it will communicate in a second to a body falling freely $= 32$,
h is the depth of the fluid in repose,
k is the height of the crest of the wave above the plane of repose.

The velocities of waves of the first order in channels of different depths are, therefore, as the square roots of the depth of these channels.

Nevertheless, when the height of one of the waves is considerable compared with the depth of the channel, a high wave in the shallower channel may move faster than a lower wave in a deeper channel; provided only the excess in height of the higher wave be greater than the difference of depth of the channels; in short, that wave will move fastest in a given channel whose crest is highest above the bottom of the channel, and in channels of different depths waves may be propagated with equal velocities, provided only the sum of the height of wave and standing depth of channel amount to the same quantity.

TABLE II.

History of a Solitary Wave of the First Order, from observation.

Depth of fluid in repose in the channel 5·1 inches.
Breadth of the channel 12 inches; the form rectangular.
Volume of generating column 445 cubic inches.

Column A is the observed height of the crest of the wave in inches above the bottom of the channel.

Column B is the observed height of the crest of the wave in inches above the surface of the water in repose.

Column C is the time in seconds occupied in traversing the distances in column D.

Column D is the spaces traversed by the wave in feet previous to each observation of time.

Column E is the velocity of the wave through each length of 40 feet deduced from observation.

Column F is the velocity deduced from the formula $\sqrt{g(h+k)} = v$.

A.	B.	C.	D.	E.	F.	G.
6·44	1·34	0·0	0	0·0		
6·41	1·31	9·5	40	4·21	4·15	− ·06
6·35	1·25	19·0	80	4·21	4·13	− ·08
6·26	1·16	29·0	120	4·0	4·11	+ ·11
6·16	1·06	39·0	160	4·0	4·08	+ ·08
6·05	0·95	49·0	200	4·0	4·04	+ ·04
5·86	0·76	59·0	240	4·0	3·99	− ·01
5·83	0·73	69·0	280	4·0	3·96	− ·04
5·76	0·66	79·5	320	3·81	3·94	+ ·13
5·68	0·58	89·5	360	4·0	3·91	− ·09
5·63	0·53	100·0	400	3·81	3·89	+ ·08
5·52	0·42	110·5	440	3·81	3·86	+ ·05
5·51	0·41	121·0	480	3·81	3·84	+ ·03
5·47	0·37	131·5	520	3·81	3·83	+ ·02
5·42	0·32	142·0	560	3·81	3·82	+ ·01
5·37	0·27	152·5	600	3·81	3·80	− ·01
5·36	0·26	163·0	640	3·81	3·79	− ·02
5·32	0·22	173·5	680	3·81	3·78	− ·03
5·31	0·21	184·0	720	3·81	3·77	− ·04
5·29	0·19	195·0	760	3·63	3·77	+ ·14
5·27	0·17	205·5	800	3·81	3·76	− ·05
5·26	0·16	216·5	840	3·63	3·75	+ ·12
5·25	0·15	227·5	880	3·63	3·75	+ ·12
5·24	0·14	237·5	920	4·0	3·75	− ·25
5·23	0·13	248·5	960	3·63	3·74	+ ·11
5·22	0·12	259·5	1000	3·63	3·74	+ ·11
5·20	0·10	270·0	1040	3·81	3·73	− ·08
5·19	0·09	281·0	1080	3·63	3·73	+ ·10
5·19	0·09	291·5	1120	3·81	3·73	− ·08
5·18	0·08	302·5	1160	3·61	3·72	+ ·11
						+1·36
						−0·84
						+0·52
					Mean .	+0·018

History of a Solitary Wave of the First Order.—In the accompanying table is given a history of the progress of a wave from its genesis through a range of 1160 feet, and during a period of 302 seconds. This wave was generated in the manner already described, by the addition of a volume of 445 cubic inches to the fluid at one extremity of the channel. The fluid in repose had a depth of 5·1 inches, and the wave generated had a height of 1·34 inch above the plane of repose, thus making the whole depth reckoned from the crest of the wave to the bottom of the channel $=5{\cdot}1+1{\cdot}34=6{\cdot}44$ inches as the depth total. This, as successively observed, forms column A, and the simple height of the wave above the plane of repose forms column B. The height of the wave is recorded at successive distances of 40 feet, as recorded in column D, reckoning from the first observation, and the corresponding time of transit past the station of observation is given in column C. The column E gives the velocity between two successive stations as resulting from the observations C and D. In order to compare these observations with the formula $v=\sqrt{g(h+k)}$, g is taken at the value 32·1908 feet, being the velocity required in one second by a body falling freely *in vacuo* in the latitude of Greenwich at the level of the sea, and $(h+k)$ is the number of inches in column A, reduced to decimals of a foot. The number resulting from these as the velocity per second which a heavy body will acquire in falling freely by gravity through a space equal to half the depth (reckoned from the crest of the wave), is that given in column F; with which the numbers in column E resulting from observation are compared, their excess or defect being set down with the signs + or − in column G.

We are thus enabled to compare the numbers given by observation E with the numbers given by formula F, and the result G shows that the coincidence is as close as the means of observation would admit. It was not possible with the chronometer then applied (although observations to fifths of a second have since been obtained) to depend upon accuracy to more minute intervals than half-seconds, and the differences in column G are precisely what we should have expected, being nearly alternately + and −, and being of nearly the same magnitude at both ends, and along the whole line of observation. The sum of the errors affected by the positive sign is + 1·36, the sum of those affected with the negative sign − 0·84, so that the whole of 29 observations give only an excess of + ·52, or a mean excess of 0·018, showing a mean excess of velocity of the observation over the velocity assigned by the formula, of 0·018 of a foot per second, being less than $\frac{1}{200}$th of the whole. Hence we are warranted in assuming, that as far as the history of this wave is concerned, the velocity is accurately represented to within $\frac{1}{200}$th part by the formula $\sqrt{g(h+k)} = v$.

Experiments on the Velocity.—In order to determine the velocity of the wave of the first order with accuracy, a series of experiments have been made upon rectangular channels, extending from 1 inch in depth and 1 foot wide, to 12 feet wide and 6 feet deep. These experiments, forming a series of thirty different depths, are given in Table III. Column A contains the depth of the water, reckoned from the crest of the wave. Column B is the height of the crest of the wave above the level of the water in repose. Column C is the velocity of the wave as observed, and in column D is given the velocity due to half the depth in column A calculated by the formula $v = \sqrt{g(h+k)}$. Columns D and C are compared, and their difference given in E, from which it results that the formula represents the experiments to within a mean error of 0·007. The results of this table leave no room to doubt that, as far as observation can settle this point, the velocity is conclusively settled, and determined to be *that due by gravity through half the depth of the fluid, reckoned from the ridge of the wave.*

TABLE III.

Determination of the Velocity of the Wave of the First Order, from observation. (See Seventh Report of the British Association, and Researches on Hydrodynamics in the Philosophical Transactions of the Royal Society of Edinburgh, 1836.)

The form of the channels was rectangular.

The breadth of the channels varied from 12 inches to 12 feet.

Column A gives the depth of the channel in inches reckoned from the top of the wave.

Column B gives the height of the wave above the surface of the fluid in repose.

Column C is the velocity of the wave in feet per second, from observation.

Column D is the velocity of the wave calculated by formula B.
Column E is the difference between columns D and C.

A.	B.	C.	D.	E.	A.	B.	C.	D.	E.
1·0			1·63		6·9	0·7	4·29	4·30	+·01
1·05	0·05	1·64	1·67	+·03	7·0			4·33	
1·30	0·15	1·84	1·86	+·02	7·33	0·29	4·39	4·43	+·04
1·62	0·32	2·06	2·08	+·02	7·44	0·40	4·44	4·46	+·02
2·0			2·31		7·82	0·78	4·53	4·57	+·04
2·19	0·29	2·30	2·42	+·12	8·0	0·78	4·53	4·63	+·10
3·0			2·83		9·0			4·91	
3·10	0·16	2·87	2·88	+·01	10·0			5·18	
3·23	0·15	2·99	2·94	−·05	11·0			5·43	
3·84	0·92	3·24	3·21	−·03	15·0			6·34	
3·9	0·96	3·33	3·23	−·10	19·0			7·14	
3·97	0·81	3·26	3·26	·00	20·0			7·32	
4·0	0·19	3·33	3·27	−·06	21·0			7·50	
4·08	0·13	3·24	3·30	+·06	26·0			8·35	
4·20	0·13	3·33	3·35	+·02	27·0			8·51	
4·31	0·24	3·40	3·40	·00	28·0			8·66	
4·49	0·42	3·46	3·47	+·01	29·0			8·82	
4·61	0·74	3·52	3·51	−·01	30·0			8·97	
4·75	0·8	3·52	3·56	+·04	35·0			9·68	
5·0			3·66		42·0	3·0	10·59	10·61	+·02
5·20	0·10	3·73	3·73	·00	45·0			10·98	
5·25	0·15	3·72	3·75	+·03	50·0			11·58	
5·61	0·57	4·05	3·88	−·17	55·0			12·14	
5·82	0·72	3·90	3·95	+·05	60·0			12·68	
6·0			4·01		65·0			13·20	
6·47	0·27	4·14	4·16	+·02	70·0			13·70	
6·74	0·54	4·32	4·25	−·07	75·0	9·0	14·23	14·18	−·05
									+·66
									−·54
									+·12
								Mean.	+·004

It appeared to me at one time matter of doubt, whether waves very low in height were not somewhat slower than the velocity of the formula, and those of a large size somewhat more rapid. To determine this point, Tables IV. and V. were prepared, the former consisting of larger waves, the latter of smaller. It can scarcely be said that these tables, which are arranged exactly as the previous one, established any distinction in this respect.

To render the results of all these experiments still more appreciable, they are graphically laid down in Plate II., the stars representing the individual experiments, and the line the formula. The coincidence is satisfactory.

TABLE IV.

Velocity of Larger Waves.

A.	B.	C.	D.	E.
1·62	0·32	2·06	2·08	+·02
3·84	0·92	3·24	3·21	−·03
3·9	0·96	3·33	3·23	−·10
3·97	0·81	3·26	3·26	·00
4·49	0·42	3·46	3·47	+·01
4·52	0·56	3·47	3·48	+·01
4·61	0·74	3·52	3·51	−·01
4·75	0·8	3·52	3·56	+·04
5·61	0·57	4·05	3·88	−·17
5·80	0·7	4·0	3·94	−·06
5·82	0·72	3·90	3·95	+·05
6·75	0·5	4·13	4·25	+·12
6·86	0·61	4·21	4·28	+·07
6·9	0·7	4·29	4·30	+·01
7·82	0·78	4·53	4·57	+·04
7·84	0·8	4·43	4·58	+·15
7·87	0·83	4·53	4·59	+·06
8·0	0·78	4·53	4·63	+·10
				+·68
				−·37
				+·31
			Mean.	+·017

TABLE V.

Velocity of Smaller Waves.

A.	B.	C.	D.	E.
1·0			1·63	
1·05	0·5	1·64	1·67	+·03
1·30	0·15	1·84	1·86	+·02
2·0			2·31	
2·19	0·29	2·30	2·42	+·12
3·0			2·83	
3·10	0·16	2·87	2·88	+·01
3·23	0·15	2·99	2·94	−·05
4·00	0·19	3·33	3·27	−·06
4·08	0·13	3·24	3·30	+·06
4·20	0·13	3·33	3·35	+·02
4·31	0·24	3·40	3·40	·00
5·0			3·66	
5·20	0·10	3·73	3·73	·00
5·25	0·15	3·72	3·75	+·03
6·0			4·01	
6·40	0·15	4·04	4·14	+·10
6·47	0·27	4·14	4·16	+·02
6·74	0·54	4·32	4·25	−·07
7·0			4·33	
7·33	0·29	4·39	4·43	+·04
7·44	0·40	4·44	4·46	+·02
8·0			4·63	
				+·47
				−·18
				+·29
			Mean.	+·018

Wave of the First Order not formerly described.—Although many distinguished philosophers from the time of Sir Isaac Newton have devoted themselves to the study of the theory of waves, I have not been able to discover in their works anything like the prediction of a phenomenon such as the wave of translation or the solitary wave of the first order. The waves of the second order, or gregarious oscillations, which make their appearance in successive groups, or long and recurring series, such oscillations of the surface of the water as we notice on the sea, or are excited when the quiescent surface of a lake is disturbed by dropping a stone, and which diffuse themselves in concentric circles around the centre of derangement; these have long been familiar to naturalists, and have been studied, though with comparatively little success, by philosophers. But I have not found the phenomenon, which I have called the wave of the first order, or the great solitary wave of translation, described in any observations, nor predicted in any theory of hydrodynamics.

Unquestionably the means of making such a prediction must have existed in any sound theory. It is, I think, pretty generally admitted that Lagrange was quite successful in stating the general equations of fluid motion; so that it was only necessary to obtain complete solutions of these equations to exhibit the formulæ of all motion consistent with the maintenance of continuity of the fluid and obedience to the laws of motion and pressure. After finding the general equations for the motion of incompressible fluids in the "Mécanique Analytique," part 2, sect. ix., Lagrange says, "Voilà les formules les plus générales et les plus simples pour la détermination rigoureuse du mouvement des fluides. La difficulté ne consiste plus que dans leur intégration;" and then he adds elsewhere, "Malheureusement elles sont si rebelles, qu'on n'a pu jusqu'à présent en venir à bout que dans des cas très-limités." Indeed, ever since the publication of Euler's general formula for the motion of fluids in the Memoirs of the Academy of Sciences of Berlin, 1755, the whole phenomena of fluids in all conditions may be considered as having been represented. But the phenomena have remained there till now, locked up without any one to open, and amongst the rest I presume the wave of the first order.

There is one point, however, in which the analysis of M. Lagrange has appeared to make an approach to the representation of one of the phenomena peculiar to the wave of translation. In section xii. of part 2 of the "Mécanique Analytique," he investigates the propagation of vibrations in elastic fluids (like those of sound through the atmosphere), and obtains an equation

$$\frac{d^2\phi}{dt^2} = gh\left(\frac{d^2\phi}{dy^2} + \frac{d^2\phi}{dx^2}\right),$$

from which he afterwards deduces the well-known law that sound is propagated with a velocity (nearly) equal to that which is due to gravity, acting freely through a height equal to half the depth of the atmosphere (supposed homogeneous and of uniform density). And again, elsewhere he finds for the propagation of wave motion in a liquid in a channel with a level bottom, and a depth α, the equation

$$\frac{d^2\phi}{dt^2} = g\alpha\left(\frac{d^2\phi}{dy^2} + \frac{d^2\phi}{dx^2}\right);$$

and from the similarity of this to the former equation, he argues as follows: "Ainsi comme la vîtesse de la propagation du son se trouve égale à celle qu'un corps grave acquerrait en tombant de la moitié de la hauteur de l'atmosphère supposée homogène, la vîtesse de la propagation des ondes sera la même que celle qu'un corps grave acquerrait en déscendant d'une hauteur égale à la moitié de la profondeur de l'eau dans le canal."

Had this result been of the same general nature with the original equations from which it is deduced, we should have been able to assign to the analysis of M. Lagrange the honour of having predicted in 1815 the wave of the first order, never distinctly recognised by observation till 1834. Unhappily the nature of his investigation precludes us from doing so, and he goes on himself to admit that this conclusion will only apply to such waves as are infinitely small, and agitate the water to a very small depth below the surface. "On pourra toujours employer la théorie précédente, si on suppose que dans la formation des ondes l'eau n'est ébranlée et remuée qu'à une profondeur très-petite." The wave of the first order bears as its characteristics the observed phenomena, that the agitation does extend below the surface to the very bottom of the channel, where it is quite as great as at the surface, and that its oscillations are large. The essential conditions of Lagrange's analysis being that the oscillation is minute, and that the agitation of the fluid is confined to the surface, we are precluded from the application of his formula to the wave of the first order.

I have been led to speak thus fully of M. Lagrange's solution, because his result is the only one that offers a tolerable approximation to the representation of the velocity of the wave of the first order. I do not find in the results obtained by M. Poisson in his "Theory of Waves," any result that represents the phenomena of this wave, although he shows that the solution of Lagrange cannot either mathematically or physically be applied to considerable depths. Nearly all of them seem to apply only to the phenomena of the fluid in the vicinity of the initial disturbance. The supposed method of genesis is one also which precludes the existence of the wave of the first order.

The greater part of the investigations of M. Poisson and of

M. Cauchy under the name of wave theory, are rather to be regarded as mathematical exercises than as physical investigations; but an account of what has been accomplished in this way by them, and by M. Laplace, may be found in the excellent Reports of Mr. Challis in the Transactions of the British Association, and in the treatise of MM. Weber.*

* I think it right in this place to mention, with such distinction as I am able to bestow, a very valuable treatise on waves, which was published nearly twenty years ago in Leipsic, by the brothers Ernest H. Weber and William Weber, entitled "Wellenlehre auf Experimente gegründet, oder über die Wellen tropfbarer Flüssigkeiten mit Anwendung auf die Schall- und Licht-Wellen, von den Brüdern Ernst Heinrich Weber, Professor in Leipzig und Wilhelm Weber in Halle. Mit 18 Kupfertafeln. Leipzig, bei Gerhard Fleischer, 1825." The work is distinguished by more than the usual characteristics of German industry in the collection of materials, and contains nearly all that has ever been written on waves since the time of Newton, and as a book of reference alone is a valuable history of wave research. To this synopsis of the labours of others is appended a valuable series of experiments by the Messrs. Weber themselves, contrived with much ingenuity, and conducted with apparently a high degree of accuracy, designed to illustrate, extend, contradict, or confirm the various theories that have been advanced. I have been disposed to regret that this excellent book did not reach me till long after my own researches had advanced far towards completion. But if it had done so, it might have diverted me from my own trains of research. As the subject now stands, it so happens that their labours and mine do not in the least degree supersede or interfere with each other. Our respective works may be rather reckoned as supplementary the one to the other, inasmuch as a great part of what they have done I have not attempted, and the most part of what I have done will not be found in any part of their work. Of the existence of my great solitary wave of the first order they were not aware, and although I am now able to recognise its influence on their results, yet owing to the nature of their experiments, it was not likely they should recognise its existence, much less could they examine its phenomena.

The following passages serve to show that the Messrs. Weber had never recognised the existence of my solitary wave of the first order. They say in Abschnitt IV. Art. 87—

"Waves make their appearance as heights and hollows upon the surface of the liquid, one part being raised above the level surface, and another part sunk below it; hence the height may be called the wave-ridge, and the depression the wave-hollow. These wave-ridges and wave-hollows never come singly, but always connected with one another. This is the reason why we do not call the wave-ridge by itself alone *a wave*, nor the wave-hollow by itself alone a wave, but simply the two conjoined." Art. 89. "The sum of the breadths of one wave-ridge and of its companion wave-hollow, is called the breadth of a wave." Art. 101. "But never in nature appears a wave-ridge unconnected with a wave-hollow, nor in like manner any wave-hollow without its companion wave-ridge. Also from this reason it follows that we can never have, during wave-motion, a particle of the fluid moved forward in its path without immediately before or after having a contrary motion also; nor backwards, without also its path being reversed."

Their observations on the larger class of waves are ingeniously contrived, carefully observed, and faithfully recorded, but lose much of the value as the basis of calculation and of general laws from the following circumstances:—1st, the narrowness of the channel; that in which the greater number of observations was made, being only 6·7 lines wide; from this cause so great an influence was produced by the adhesion of the sides as seriously to interfere with the phenomena, which ought not therefore to be considered as the phenomena of perfectly free fluids; 2d, the shortness of the channels; the longest having a depth of 2 feet and only 6 feet of length; in this case an observation of the wave of the first order was impossible; and when we add that the wave genesis was in general produced by the descent of a water column of great height, it was impossible

Having ascertained that no one had succeeded in predicting the phenomenon which I have ventured to call the wave of translation, or wave of the first order, to distinguish it from the waves of oscillation of the second order, it was not to be supposed that after its existence had been discovered, and its phenomena determined, endeavours would not be made to reconcile it with previously existing theory, or in other words, to show how it ought to have been predicted from the known general equations of fluid motion. In other words, it now remained to the mathematician to predict the discovery after it had happened, *i.e.*, to give an *à priori* demonstration *à posteriori*.

Theoretical Results subsequent to the publication of the Author's Investigations.—Since the publication of my former observations on the wave of the first order, two attempts have been made to elicit from the wave theory, as developed by Poisson, &c., results

that in the short period of wave transit the phenomena could attain a condition of uniformity favourable to accurate observation, one second and a fraction of a second being the whole period of an observation, and it being necessary to observe accurately to at least one-twentieth of a second, the results possess little value as measures of the phenomena. In my experiments we found that the first observations immediately after the wave genesis were the least accurate and the least valuable, and these are the *only* observations employed by MM. Weber in their larger wave observations. Further, as they did not recognise at all the possibility of the existence of the solitary wave of the first order, nor the difference of its phenomena from the negative waves, nor the distinction of waves into separate first and second orders, they have mingled together the observations and phenomena of both. Thus have they failed to recognise the existence of the law of the velocity which I have elicited.

Nevertheless, their observations are very valuable, and furnish interesting information to one already master of my observations. In their very deviations from the laws exhibited by my observations, they become instructive as manifesting and enabling us to measure the amount of those interfering influences which diminished the value of their experiments when taken by themselves. For this purpose I have taken some of their experiments and placed them beside the results of mine; the effects of adhesion to the sides, and of more or less perfect fluidity, are well manifested in the difference of the results. It is, however, to be remembered that in point of accuracy and precision, and also of weight, the shortness of period and path in their observations diminish their value.

These remarks, which I make with perfect deference, are designed to apply only to the large class of waves to which chiefly I have directed my attention; the observations on drooping waves, and all those made with reference to the phenomena of light and sound, are to be exempted from these remarks. I desire that my experiments should enhance rather than derogate from the value of those of my estimable predecessors, and I wish rather by these statements to make an apology to them for having arrived at different conclusions, by showing that the methods I chanced to light upon, and the circumstances in which I observed, were more favourable than those which they happened to employ. I only aspire to having brought to a more favourable conclusion what they had most meritoriously begun under circumstances less propitious; my having arrived at different conclusions is probably more owing to the chance of my being ignorant of their methods when I began, and alighting by chance upon better; for had I known of their elegant apparatus at first, it is not improbable that I should have been satisfied to adopt what so much ingenuity had contrived, and so failed to extend the subject beyond the conclusions they had attained.

capable of such physical interpretations as should represent the phenomena of that order.

The first of these investigations is that of Mr. Kelland in the Edinburgh Philosophical Transactions. This valuable and elegant investigation deduces theoretically, from the general equations of fluid motion, on the hypothesis of parallel sections, and of oscillations of the general form of the curve of sines, the following value for the velocity of a wave :—

$$c^2 = \frac{g}{\alpha} \cdot \frac{e^{ah} - e - {}^{ah}}{c^{ah} + e - {}^{ah}} \div \left\{ 1 - \epsilon a \frac{e^{ah} - e^{ah}}{e^{ah} + e^{ah}} \right\}, \quad . \quad . \quad . \quad [\text{C.}]$$

ϵ being the semi-elevation, h the depth in repose, λ the length of the wave, c the velocity of transmission.

This expression gives values for the velocity of the wave which Mr. Kelland has himself compared with my experiments as follows :—

Theoretical value when $h = 3{\cdot}97$ and $2e = 0{\cdot}53$, is $c = 2{\cdot}8693$
Observed value $c = 3{\cdot}38$,

showing the error in defect $= -\dfrac{1}{5\frac{1}{2}}$ or $-\dfrac{2}{11}$ of the whole theoretical velocity.

Another example :

Theoretical value (when $h = 1$ and $2e = 0{\cdot}3$) $c = 1{\cdot}547$
Observed value $c = 1{\cdot}8$,

showing the error in defect $= -\frac{1}{6}$ of the whole theoretical velocity.

Again,

Theoretical value (when $h = 7{\cdot}04$ and $2e = 0{\cdot}89$) $c = 4{\cdot}0$
Observed value $c = 4{\cdot}6$,

showing the error in defect $= -\frac{1}{8}$ of the whole theoretical velocity.

I think it due to Mr. Kelland to say, that notwithstanding all the anxiety for success which naturally exists in the mind of one who has bestowed much time and talent on perfecting, as he has done, an elegant theory, he has not yielded to the temptation of twisting his theory to exhibit some apparent approximation to the facts, nor distorted the facts to make them appear to serve the theory, a proceeding not without precedent; but he has candidly stated the discrepancy, and says, "My solution can only be regarded as an approximation, nor does it very accurately agree with observation." This is a candour which cannot be too highly valued, and can only be justly appreciated by those who have, as I have, after working at a favourite theory, it may be for months and years, found it necessary to abandon it, and make the sacrifice for the sake of truth with readiness and candour.

Mr. Airy has followed Mr. Kelland over the same ground, in an elaborate paper on waves in the "Encyclopedia Metropolitana," published since the greater part of this Report was ready for the press. This paper I have long expected with much anxiety, in the hope that it would furnish a final solution of this difficult problem, or at least tend to reduce the number and extent of the unhappy discrepancies between the wave-prediction and the wave-phenomena, a hope justified by the reputation and position of the author, as well as by the clear views and elegant processes which characterise some of his former papers.

Mr. Airy has obtained for the velocity of a wave, an expression of a form closely resembling that which Mr. Kelland had previously obtained, viz.

$$c^2 = \frac{g}{m} \cdot \frac{\varepsilon^{mk} - \varepsilon^{-mk}}{\varepsilon^{mk} + \varepsilon^{-mk}} \quad . \; . \; . \; . \; . \; . \quad \text{[D.]}$$

From the resemblance of this form of expression to the form previously given by Mr. Kelland, we are prepared for the conclusion that Mr. Airy has advanced in this direction little beyond his predecessor. And we accordingly find that a theory of the wave of the first order, accurately representing this characteristic phenomenon, is still wanting, a worthy object for the enterprise of a future wave-mathematician.

I have already stated that I have found, that by introducing the element of the wave's height into Lagrange's formula, I get the expression

$$v = \sqrt{g(h + k)},$$

and that I find it represents with great accuracy the characteristic velocity of the wave of the first order. As, however, Mr. Airy appears to intimate to his readers that his own formula is as close an approximation to my experiments as the nature of these experiments will warrant, I have thought it necessary to make a complete re-examination of my experiments, and to make a laborious comparison of the phenomena discussed after the best modern methods employed in inductive philosophy; the results of these discussions I have presented in a series of graphic representations, which will enable the reader at once to attain a sound conclusion on the question, whether the formula Mr. Airy has adopted, or that which I have always used, more truly represents the phenomena.

In the following table, E represents the velocity of the wave of the first order as taken from my observations by Mr. Airy himself. I have placed beside these results of experiments, the number given in column F, by the formula which I use to represent them. In the next four columns are Mr. Airy's numbers, calculated by himself, according to four different formulæ, which he appears here to have applied as a sort of tentative process for the purpose of selecting the one which should prove on trial least defective. I have next given five columns, which exhibit the results of comparing the phenomena of experiment with the results of the formulæ. The first of these columns represents the defects of my formula, the others those of Mr. Airy's.

The results of the first table are as follows:—

The errors of Mr. Airy's first column amount to . .	2635
The errors of Mr. Airy's second column amount to .	1994
The errors of Mr. Airy's third column amount to . .	1674
The errors of Mr. Airy's fourth column amount to .	1680
The errors of mine amount to	406

The greatest error of Mr. Airy's first column is .	809
The greatest error of Mr. Airy's second column is	690
The greatest error of Mr. Airy's third column is .	463
The greatest error of Mr. Airy's fourth column is	575
The greatest error of mine is	87

The results of the second table are as follows:—

The errors of Mr. Airy's first column amount to . .	6157
The errors of Mr. Airy's second column amount to .	3350
The errors of Mr. Airy's third column amount to . .	3226
The errors of Mr. Airy's fourth column amount to .	2274
The errors of mine amount to	447

The greatest error of Mr. Airy's first column is .	911
The greatest error of Mr. Airy's second column is	689
The greatest error of Mr. Airy's third column is .	473
The greatest error of Mr. Airy's fourth column is	480
The greatest error of mine is	122

TABLE VI.

Small Waves.

Column A is a mean height of wave crest.
„ B the selected examples from which A is taken.
„ C the depth of the fluid in repose.
„ D the height of the wave.
„ E the velocity of the wave observed.

(B–E: As taken from my experiments by Mr. Airy.)

„ F the velocity of the wave as given by my formula.
„ G the velocity of the wave as given by Mr. Airy's first formula.
„ H the velocity of the wave as given by Mr. Airy's second formula.
„ K the velocity of the wave as given by Mr. Airy's third formula.
„ L the velocity of the wave as given by Mr. Airy's fourth formula.
„ F′ the difference between observation and my formula.
„ G′ the difference between observation and Mr. Airy's first formula.
„ H′ the difference between observation and Mr. Airy's second formula.
„ K′ the difference between observation and Mr. Airy's third formula.
„ L′ the difference between observation and Mr. Airy's fourth formula.

A.	B.	C.	D.	E.	F.	G.	H.	K.	L.
1·075	1·05 and 1·10	1·000	0·075	1·670	1·697	1·629	1·689	1·803	1·747
1·3	1·3	1·150	0·150	1·810	1·867	1·744	1·854	2·057	1·958
3·17	3·09—3·23	2·963	·207	2·860	2·915	2·702	2·795	2·972	2·885
3·36	3·32 and 3·40	3·080	·280	2·960	3·002	2·747	2·869	3·099	2·986
4·16	4·0 —4·31	3·903	·256	3·310	3·340	3·016	3·114	3·300	3·208
5·34	5·20—5·5*	5·088	·252	3·758	3·784	3·303	3·384	3·540	3·463
6·52	6·4 —6·65	6·220	·304	4·094	4·181	3·495	3·579	3·742	3·662
7·51	7·42—7·7	7·040	·474	4·406	4·488	3·597	3·716	3·943	3·831

* Excluding 5·21.

Differences.

F′.	G′.	H′.	K′.	L′.
+ ·027	− ·041	+ ·019	+ ·133	+ ·077
+ ·057	− ·066	+ ·044	+ ·247	+ ·148
+ ·055	− ·158	− ·065	+ ·112	+ ·025
+ ·042	− ·213	− ·091	+ ·139	+ ·026
+ ·030	− ·294	− ·196	− ·010	− ·102
+ ·026	− ·455	− ·374	− ·218	− ·295
+ ·087	− ·599	− ·515	− ·352	− ·432
+ ·082	− ·809	− ·690	− ·463	− ·575
+ ·406	− 2·635	− 1·931	− 1·043	− 1·404
		+ ·063	+ ·631	+ ·276
·406	− 2·635	1·994	1·674	1·680

TABLE VII.

Large Waves.

Columns A, B, C, &c., correspond to those in Table VI.

A.	B.	C.	D.	E.	F.	G.	H.	K.	L.
1·20	1·20	1·000	0·200	1·760	1·794	1·629	1·785	2·061	1·928
1·62	1·62	1·300	·320	2·060	2·083	1·858	2·072	2·446	2·267
2·19	2·19	1·900	·290	2·300	2·422	2·217	2·380	2·677	2·533
3·38	3·35—3·41	2·960	·420	3·010	3·010	2·701	2·887	3·225	3·061
3·55	3·5 —3·61	3·020	·532	3·080	3·085	2·724	2·954	3·368	3·168
3·83	3·69—3·97	3·007	·830	3·252	3·204	2·719	3·072	3·677	3·388
4·53	4·4 —4·75	3·910	·625	3·505	3·485	3·018	3·250	3·671	3·467
5·21	5·21	3·870	1·340	3·820	3·738	3·007	3·488	4·293	3·911
5·76	5·61—5·82 ·	5·070	0·692	3·970	3·930	3·300	3·518	3·917	3·723
6·24	6·15—6·40	5·080	1·160	4·170	4·090	3·302	3·659	4·286	3·985
6·69	6·69—7·20	6·034	0·823	4·262	4·234	3·468	3·697	4·117	3·912
7·83	7·74—8·0	6·946	0·884	4·497	4·582	3·586	3·808	4·216	4·017

Differences.

F′.	G′.	H′.	K′.	L′.
+ ·034	− ·131	+ ·025	+ ·301	+ ·168
+ ·023	− ·202	+ ·012	+ ·386	+ ·207
+ ·122	− ·083	+ ·080	+ ·377	+ ·233
+ ·00	− ·309	− ·123	+ ·215	+ ·051
+ ·005	− ·356	− ·126	+ ·288	+ ·088
− ·048	− ·533	− ·180	+ ·425	+ ·136
− ·020	− ·487	− ·255	+ ·166	− ·038
− ·082	− ·813	− ·332	+ ·473	+ ·091
− ·040	− ·670	− ·452	− ·053	− ·247
− ·080	− ·868	− ·511	+ ·116	− ·185
− ·028	− ·794	− :565	− ·145	− ·350
+ ·085	− ·911	− ·689	− ·281	− ·480
− ·298	− 6·157	− 3·233	+ 2·747	+ ·974
+ ·269		+ ·117	− ·479	−1·300
·567	6·157	3·350	3·226	2·274

The conclusion which Mr. Airey deduces from this comparison is somewhat surprising, "We think ourselves fully entitled to conclude from these experiments that the theory (Mr. Airey's) is entirely supported!" This conclusion being so completely the opposite of that to which we should be led on the same grounds, it has appeared necessary to make a still more complete re-examination and discussion of all the experiments in our possession, to see whether from any or the whole of them there should appear to be any ground for a conclusion so contrary to the apparent phenomena.

I have, therefore, directed the whole of the experiments to be re-discussed.* They are graphically represented in the diagrams on Plates II. and III., which, and the description, the reader is requested to examine carefully. The result of the whole is, that there is an irresistible body of evidence in favour of the conclusion that Mr. Airy's formulæ do not present anything like even a plausible representation of the velocity of the wave of the first order, and that the formula I have adopted does as accurately represent them as the inevitable imperfections of all observations will admit. It is deeply to be deplored that the methods of investigation employed with so much knowledge, and applied with so much tact and dexterity, should not have led him to a better result.

*** For the accuracy and good faith with which these discussions were all conducted, I am indebted to my valued assistant, Mr. I. Currie.**

TABLE VIII.

Re-discussion of the Observations by the Method of Curves.

The observations of height and time were laid down on paper, as shown in Plate III. (see description), each star representing an individual observation of height or time. The curves being drawn through among the observations, were taken to represent the *corrected observations*, and the *velocity* was then deduced from the corrected observation of *time* and *height*. The table consists of results of this process.

Column A gives the corrected depth in inches $(h+k)$ of my formula.

Column B gives the corrected time in seconds employed in describing 40 feet.

Column C gives the derived velocity of the wave.

Column D gives the characteristic number of the individual wave as observed (see former Report).

These results are compared with my formula in Plate III.

A.	B.	C.	D.
	s.	ft.	
3·20	14·0	2·85	26
3·20	13·9	2·87	
3·20	13·7	2·92	
3·21	13·6	2·94	
3·22	13·5	2·96	
3·24	13·4	2·98	
3·27	13·3	3·00	
3·30	13·2	3·03	
3·35	13·1	3·05	
3·40	13·0	3·07	
3·45	12·9	3·10	
3·53	12·8	3·12	
3·61	12·7	3·15	
3·72	12·6	3·17	
3·84	12·5	3·20	
3·97	12·4	3·22	
3·97	12·5	3·20	2
4·00	12·3	3·25	
4·03	12·1	3·30	
4·07	12·0	3·33	
4·12	11·9	3·36	
4·17	11·9	3·36	
4·22	11·8	3·39	
4·28	11·8	3·39	
4·34	11·7	3·41	
4·42	11·7	3·41	
4·49	11·6	3·44	
4·04	12·2	3·27	3
4·06	12·1	3·30	
4·08	12·0	3·33	
4·11	11·9	3·36	
4·15	11·8	3·39	
4·20	11·8	3·39	

A.	B.	C.	D.
	s.	ft.	
4·24	11·8	3·39	3
4·30	11·7	3·41	
4·36	11·7	3·41	
4·43	11·7	3·41	
4·05	12·3	3·25	1
4·06	12·2	3·27	
4·08	12·2	3·27	
4·10	12·1	3·30	
4·12	12·0	3·33	
4·14	12·0	3·33	
4·17	11·9	3·36	
4·20	11·9	3·36	
4·23	11·9	3·36	
4·27	11·8	3·39	
4·32	11·8	3·39	
4·36	11·7	3·41	
4·42	11·7	3·41	
4·48	11·6	3·44	
4·26	12·2	3·27	8
4·28	12·1	3·30	
4·30	12·0	3·33	
4·32	11·9	3·36	
4·35	11·8	3·39	
4·38	11·7	3·41	
4·42	11·6	3·44	
4·46	11·5	3·47	
4·51	11·5	3·47	
4·57	11·4	3·50	
4·63	11·3	3·54	
4·70	11·3	3·54	
4·77	11·2	3·57	
4·85	11·2	3·57	
4·95	11·1	3·60	

A.	B.	C.	D.
	s.	ft.	
5·07	11·0	3·63	8
5·18	11·0	3·63	45
5·18	10·9	3·66	
5·19	10·9	3·66	
5·20	10·9	3·66	
5·21	10·8	3·70	
5·22	10·8	3·70	
5·23	10·8	3·70	
5·25	10·7	3·73	
5·27	10·7	3·73	
5·29	10·7	3·73	
5·31	10·7	3·73	
5·33	10·6	3·77	
5·35	10·6	3·77	
5·38	10·5	3·81	
5·41	10·5	3·81	
5·44	10·4	3·84	
5·48	10·4	3·84	
5·52	10·3	3·88	
5·57	10·3	3·88	
5·63	10·2	3·92	
5·70	10·2	3·92	
5 78	10·15	3·94	
5·87	10·1	3·96	
5·97	10·0	4·0	
6·10	10·0	4·0	
6·2	10·0	4·0	
6·29	9·95	4·02	
6·37	9·9	4·04	
6·43	9·8	4·08	
6·38	9·95	4·02	50
6·39	9·9	4·04	
6·40	9·8	4·08	

A.	B.	C.	D.
	s.	ft.	
6·41	9·8	4·08	50
6·42	9·8	4·08	
6·44	9·7	4·12	
6·46	9·7	4·12	
6·48	9·7	4·12	
6·51	9·7	4·12	
6·54	9·6	4·16	
6·57	9·6	4·16	
6·60	9·5	4·21	
6·63	9·5	4·21	
6·68	9·5	4·21	
6·73	9·5	4·21	
6·78	9·4	4·25	
6·83	9·4	4·25	
6·89	9·4	4·25	
6·95	9·4	4·25	
7·02	9·4	4·25	
7·19	9·3	4·30	
6·70	9·16	4·16	49
6·81	9·5	4·21	
6·95	9·4	4·25	
7·10	9·3	4·30	
7·30	9·2	4·34	
7·52	9·1	4·39	
7·66	9·0	4·44	
6·71	9·4	4·25	48
6·79	9·3	4·30	
6·92	9·2	4·34	
7·20	9·1	4·39	
7·54	9·0	4·44	
7·75	8·9	4·49	
7·82	8·8	4·54	

TABLE IX.

Velocity due to a Wave of the First Order.

Obtained from the re-discussion of the experiments as described above.

Column A gives the depth in inches reckoned from the wave crest.

Column B gives the observed time of describing 40 feet,* the observations thus marked being over half that space.

Column C gives the observed velocity.

Column D is a reference to the ordinal number of the wave observed.

The close approximation of these velocities of observation with the numbers of the formula, proves at once the accuracy of the one and the truth of the other.

A.	B.	C.	D.	A.	B.	C.	D.	A.	B.	C.	D.	A.	B.	C.	D.
	s.	ft.			s.	ft.			s.	ft.			s.	ft.	
1·5	10·1*	1·98	35	4·0	12·5	3·20	3	4·5	11·5	3·47	15	5·5	10·5	3·80	46
2·0	8·7*	2·30	36	4·0	11·8	3·39	25	4·5	12·0	3·33	17	6·0	9·9	4·04	43
2·5	7·2*	2·77	36	4·0	6·0*	3·33	37	4·5	11·7	3·42	19	6·0	10·0	4·0	45
2·5	8·4*	2·38	35	4·0	6·1*	3·27	38	4·5	12·3	3·25	23	6·0	10·0	4·0	46
3·0	7·5*	2·66	40	4·0	6·0*	3·33	39	5·0	11·1	3·60	8	6·5	9·5	4·21	46
3·0	7·4*	2·70	41	4·0	6·2*	3·22	40	5·0	11·4	3·50	9 and 10	6·5	9·8	4·08	49
3·5	13·7	2·92	25	4·5	11·5	3·47	1	5·0	10·9	3·67	15	6·5	9·7	4·12	50
3·5	12·8	3·12	26	4·5	11·6	3·45	2	5·0	10·7	3·73	17	7·0	9·3	4·3	46
3·5	6 4*	3·12	37	4·5	12·0	3·33	4	5·0	10·9	3·67	19	7·5	9·2	4·35	53
3·5	6·2*	3·22	38	4·5	11·8	3·39	5	5·0	11·2	3·57	22	7·5	9·0	4·44	55
3·5	6·5*	3·07	40	4·5	11·4	3·50	6	5·0	11·1	3·60	23	8·0	8·9	4·49	51
3·5	6·5*	3·07	41	4·5	11·5	3·47	7	5·5	11·0	3·63	9 and 10	8·0	8·7	4·6	52
3·5	12·8	3·12	42	4·5	11·5	3·47	8	5·5	10·6	3·77	43	8·0	8·6	4·65	54
4·0	12·3	3·25	2	4·5	11·5	3·47	13	5·5	10·3	3·88	45	8·0	8·9	4·49	55

The Magnitude and Form of the Wave of the First Order.—This is one of the subjects to which, since the date of the former Report, I have devoted a good deal of attention. The exact determination of the dimensions and form of the wave, although at first sight it may seem simple enough, is not without peculiar difficulties. When it is observed that the two extremities of the wave are vertices of curves of very small curvature tangent to the plane of repose, it will be understood how difficult it is to detect the place of contact with precision. A variety of methods have been tried : reflection of an image from the surface, tangent points applied to the surface so as to be observed simultaneously at both ends of the wave, and the self-registration of a float moved by the wave have all been tried with various success. On the whole, however, the most perfect observations have been obtained by a very simple autographic method, in which it was contrived that the wave should leave its own outline delineated on the surface without the intervention of any mechanism.* The method was simply this: a dry smooth surface was placed over the surface of the water in the channel, with such arrangements that it could be moved along with the velocity of transmission of the wave, and at the instant of observation it was pushed vertically down on the wave, and raised out again without sensibly disturbing the water; the surface when brought out, brought with it a moist outline of the wave, which was immediately traced by pencil, and afterwards transferred to paper. I have given a few of these autographic types of the wave in Plates IV. and V., the engravings being precise copies of the lines as drawn by the wave itself.

Another method of obtaining an autographic representation of waves of the first order was this. Two waves were generated at opposite ends of the same channel at given instants of time, so that by calculating their velocities they should both reach a given spot at the same instant; here a prepared surface was placed, and as one passed over the other it left a beautiful outline of the excess in height of each point of one wave above the summit height of the other. These forms are not identical with those of the same wave moving along a plane surface, but as true registers of actual phenomena they are interesting.

The results of all my observations on this subject are as follows:—

That the wave of the first order has a definite *form and magnitude* as much characteristic of it as the uniform velocity with which it moves, and depending like that velocity only on the depth of the fluid and the height of the wave crest.

That this wave-form has its surface wholly raised above the level of repose of the fluid. This is what I mean to express by calling this wave *wholly positive.* I apply the word negative to another kind of wave whose surface exhibits a depression below the surface of repose. The wave-proper of the first order is wholly positive.

* I find that I am not the first person who employed an apparatus of this sort. MM. Weber employed a powdered surface to register the form of agitated mercury, the fluid rubbing off the powder.

The simple elementary wave of the first order assumes a definite *length equal to about six times the depth of the fluid below the plane of repose.* When the height of the wave is small the length does not sensibly differ from that of the circumference of a circle whose radius is the depth of the fluid: or h being the depth of the fluid in repose, the length of the wave is represented by the quantity $2\pi h$, π being the number 3·14159, we may use this notation,

$$\lambda = 2\pi h \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{E.}$$

The length, therefore, increases with the depth of the fluid directly, being equal to about 6·28 times the depth. The length does not, like the velocity of the wave, increase with the height of the wave in a given depth of fluid. On the contrary, the length appears to diminish as the height of the wave is increased, and the length of the wave when thus corrected is

$$\lambda = 2\pi h - \alpha \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{F.}$$

The value of α will be afterwards examined.

The form of the wave surface when not large is a surface of single curvature, the curvature being in the longitudinal and vertical planes alone, and the curve is the curve of sines, or rather of versed sines, the horizontal ordinates of which vary as the arc and the vertical ordinates, as the versines of a circle whose radius is the depth of the fluid in repose, $2\pi h$ being the length of the wave, and $\frac{2\pi}{m}$ an arc of that circle $= \theta$. We have for the equation of the wave curve,

$$\begin{aligned} x &= h\theta \\ y &= \tfrac{1}{2}k.\ \text{versin}\ \theta \quad . \quad . \quad . \quad . \quad . \quad \text{G.} \end{aligned}$$

the height of the wave being denoted by k, reckoned above the plane of repose of the surface of the fluid.

The height of the wave above the surface of the water in repose may increase till it be equal to the depth of the fluid in repose. When it approaches this height it becomes acuminate, finally cusped, and falls over breaking and foaming with a white crest. The limits of the wave height are, therefore,

$$k = 0, \text{ and } k = h \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{K.}$$

that is to say, the height of the wave may increase from O to k, but can never exceed a *height* above the level of repose equal to the *depth* of the fluid in repose; that is, the height total reckoned from the bottom is never greater than twice the depth of the fluid in repose.

The absolute Motions of each Water-Particle during Wave-Transmission.—This is one of the subjects on which, prior to last Report, I had not made a sufficient number of observations to enable me to make a full report. The methods I had employed for such observations as I had then already made, were the observation of the motions of small particles visible in the water of the same, or nearly the same specific gravity with water, or small globules of wax connected to very slender stems, so as to float at required depths. The motions of these were observed from above, on a minutely divided surface on the bottom of the channel, and from the side through glass windows, themselves accurately graduated, the side of the channel opposite to the window being covered with lines at distances precisely equal to those on the window and similarly situated. These methods are the only methods of observation I have found it useful to employ, but I have now increased the number and variety of the observations sufficiently to enable me to adduce the conclusions hereinafter following, as representing the phenomena as far as their nature will admit of accurate observation.

It is characteristic of waves that the *apparent motion visible on the surface* of the water is of one species, while the *absolute motion of the individual particles* of the water is very different. In reference to all the species of waves this is true, both as regards the velocity and nature of the motion; nevertheless the one is the immediate cause or consequence of the other. In the case of the wave of the first order, the visible motion of the wave form along the surface of the water may be called the *motion of transmission;* the actual motion of the particles themselves is to be distinguished as the *motion of translation.*

We infer the motions of the individual wave particles from those of visible small bodies floating in the water; any minute particle floating on the surface will sufficiently indicate the motion of the water particles about it, and the motion of deeper particles may be conveniently observed in the case of waves of the first order, by using the little globules of wax already mentioned; these small globules may be so made as to float permanently at any given depth, yet they will be visibly affected by very minute forces.

In this way the following observations were made:

Absolute Motion of Translation.—The phenomenon of translation characteristic of the wave of the first order, and which we have used as its distinguishing appellation, is to be observed as follows. Floating globules, as already described, being placed in the fluid, and their positions being noted with reference to the sides and bottom of the channel, let a wave of the first order be transmitted along the fluid; it is found that the effect of this transmission is to lift each of the floating particles, and similarly, therefore, the water particles themselves, out of their positions, and to transfer them permanently forward to new positions in the channel, and in these new positions the particles are left perfectly at rest, as in their original places in the channel.

The measure or range of translation is just equal to that which would result from increasing the column of water in the channel behind the wave by a given quantity, and diminishing the column anterior to the particles by the same quantity, that quantity being equal to the volume of the wave. That is to say, *the range of translation is simply equal to the space in length of the channel which the volume of the wave would occupy* on the level of the water in repose.

The *total effect* of having transmitted a wave of the first order along a channel, is to have moved successively every particle in the whole channel forward, through a space equal to the volume of the wave divided by the water-way of the channel.

Parallelism of Translation.—If the floating spherules before mentioned be arranged in repose in one vertical plane at right angles to the direction of transmission, and carefully observed during transmission, it will be noticed that the particles remain in the same plane during transmission and repose in the same place after transmission.

It is further found, as might be anticipated from the foregoing observations, that a thin solid plane transverse to the direction of transmission, and so poised as to float in that position, does not sensibly interfere with the motion of translation or of transmission.

The Range of Horizontal Translation is Equal at all Depths.—Vertical excursions are performed by each particle of fluid simultaneously with the horizontal translation. These diminish in extent with the distance from the bottom when they become zero.

The Path of each Water Particle during Translation lies wholly in a Vertical Plane.—It may be observed by means of the glass windows already mentioned, its surface being graduated for purposes of measurement. The path is so rapidly described that I do not think any measurements of time which I have made, nor even of paths, is *minutely* correct. The following observations are such as a practised eye with long experience and much pains has made out.

When a wave of the first order in transmission makes a transit over floating particles in a given transverse plane, the observations are as follows. All the particles begin to rise, scarcely advancing; they next advance as well as rise; they cease to rise but continue advancing; they are retarded and come to rest, descending to their original level. The path appears to be an ellipse whose major axis is horizontal and equal to the range of translation; the semi-minor axis of the elliptic path is equal to the height of the wave near the surface, and diminishes directly with the depth.

The results of these observations are, therefore, as follows:—representing by b the breadth of the channel, by h the depth of the fluid, by a the range of translation, and by v the volume of water employed in forming the waves; we have for every particle throughout the breadth and depth of the fluid

$$a = \frac{v}{bh} \quad . \quad . \quad . \quad . \quad . \quad \text{(L.)}$$

which everywhere measures the horizontal range of translation.

The range of vertical motion of each particle at the surface during translation being everywhere

$$y = k \quad . \quad . \quad . \quad . \quad . \quad \text{(M.)}$$

we have for the vertical range y' of any other particle at a depth h' below the surface,

$$y' = \frac{h'}{h} . \, k \quad . \quad . \quad . \quad . \quad \text{(N.)}$$

being directly as the height of the particle in repose above the bottom of the channel.

Also throughout the whole period of translation we have the height of a particle of the surface above its place of repose represented by

$$y = \tfrac{1}{2}k \text{ versin } \theta \quad . \quad . \quad . \quad . \quad . \quad \text{(O.)}$$

and the height of any other particle in the same vertical plane at the same place represented by

$$y' = \tfrac{1}{2}\,\frac{h'k}{h} \text{ versin } \theta \quad . \quad . \quad . \quad . \quad . \quad \text{(P.)}$$

The whole of these results are united in the following Table of wave phenomena.

Table X.

Phenomena of Wave of the First Order.

Let c be the velocity of wave transmission;
- h the depth of fluid in repose;
- k the height of wave-crest above surface of repose;
- b breadth of channel;
- v the volume of fluid constituting the wave;
- g the measure of gravity;
- a the horizontal range of translation;
- λ the wave length or amplitude;
- θ an arc $\frac{2\pi}{m}$, m being an arbitrary number;
- ψ the arc whose sine $= \frac{1}{2}$ versed sine of θ;
- π the number 3·1416;
- π' the circumference of an ellipse whose axes are given;
- x and y horizontal and vertical ordinates of wave-curve;
- x' and y' horizontal and vertical ordinates of translation-path;
- h' the height of a particle in repose, above the bottom of the channel;

Then we have

(1.) For velocity of wave transmission,

$$c = \sqrt{g(h + k)} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{B.}$$
$$= \sqrt{gh} \text{ nearly, when } h \text{ is small} \quad . \quad . \quad . \quad \text{A.}$$

(2.) For the wave length,

$$\lambda = 2\pi h - \alpha \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{F.}$$
$$= 2\pi h \text{ nearly, when } k \text{ is small} \quad . \quad . \quad . \quad \text{E.}$$

(3.) For the range of translation,

$$\alpha = \frac{v}{bh} \text{ always,}$$
$$= \pi k \text{ when } k \text{ is small,} = 2k \text{ nearly when } k \text{ is large L.}$$

(4.) For the wave form,

$$x = h\theta - x' = h\theta, \text{ when } k \text{ is small}$$
$$y = \tfrac{1}{2}k. \text{ versin } \theta \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{G}'.$$

(5.) For the path of translation,

$$\left.\begin{aligned} x' &= \alpha \text{ versin } \psi \\ y' &= \tfrac{1}{2}k \text{ versin } \theta \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{O}'.$$

and below the surface at h', $y' = \frac{h'k}{2h}$. versin θ. . . . . . P.

(6.) The limits of the value of k are as follows :—

Inferior limit $k = \text{O}$, and $k = h$ superior limit . . K.

(7.) The range of vertical motion of a particle during translation being $y = k$ at the surface, the range of vertical motion of any other particle at the height h above the bottom is

$$y' = \frac{h'}{h}k \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{N}'.$$

Geometrical Representation of the Wave of the First Order. These data enable to approximate to the exact conception of the motions of the wave particles, and the relations which the wave form and the particle path bear to each other. We may thus construct a geometrical representation of the wave motion, which, however, is to be carefully distinguished from a physical determination of its phenomena.

Let us then endeavour to follow the motion of a given particle on the surface of the fluid during the wave form transmission.

Let us take D E for the depth of the fluid. (Plate VI. fig. 3.)

Let us take C D for the height of the wave.

Let us mark off d D d' = the circumference of the circle of which D E is the radius = 6·2832 × D E. Let also semicircles be described on $c\ d$ and on $c'\ d'$ each equal to C D. Let the semicircles $c\,d$ and $c'\ d'$ and the distances d D and D d' be divided into the same number of equal parts. Let there be drawn through each division of the circles horizontal lines, and through each division of the wave lengths let there be drawn perpendiculars, meeting successively the horizontal lines in 9, 8, 7, 6, 5, 4, 3, 2, 1,—these will be points in the *curve of versed sines*, that is of the (approximate) form of the wave. If, therefore, we conceive the wave-form to move horizontally and uniformally along the line d D d', and at the same time a particle of water on the surface to rise successively to the heights 1, 2, 3, 4, 5, and fall vertically to 6, 7, 8, 9, on the diameters $c\ d$ and $c'\ d'$, then the place of the particle will always coincide with the wave curve.

This is the same form (only wholly positive) which Laplace assigns to the tide wave in the "Mécanique Céleste," tom. iii. liv. iv. chap. iii. Art. 17. "Concevons un cercle vertical, dont la circonférence en partant du point le plus bas, expriment les temps écoulés depuis la basse; les sinus verses de ces arcs, seront les hauteurs de la mer, qui correspondent à ces temps." Or as he says elsewhere, "Ainsi, la mer en s'élevant, baigne en temps égal, des arcs égaux de cette circonférence." So if we imagine a circular disc placed vertically so as to touch the surface of the water in repose, the passing wave will in successive equal times cover equal successive arcs of the circumference.

The wave is of this form when its height is small, and the deviation increases with the increase of height.

Vertical Motion of each Particle.—No more then is necessary to the exhibition of the wave curve than that every particle of the surface of the water should be made to rise and fall successively, according to the increase and decrease of the versed sines of the circle of height. Let us follow the motion of a single particle. Draw $c'\ d'$ a vertical diameter of the wave circle, suppose $C\ e\ f\ g\ h\ c'$ the successive places of the wave crest in successive equal intervals of time, 1, 2, 3, 4, 5, 6, 7, 8, 9, successive versed sines on $c\ d$ and $c'\ d'$ of equal arcs of the wave circle. When the wave centre is at C, the particle is at d'. When the wave centre is at e, the particle has risen to 1. When the wave centre has reached f, the water particle has risen to 2. When the wave has advanced to $g\ h\ c'$, &c., the water particle has risen to 3, 4, 5, &c.; and if every successive particle along the surface be conceived to perform successively a similar series of vertical motions, the surface of the water will present to the eye the visible moving wave form. Such is the simplest geometrical mode of exhibiting to the eye and of conceiving wave motion of the first order; it approximately represents the form of a wave of the first order whose height is small.

Horizontal Motion of each Particle.—This mode of representing the wave motion is inaccurate, in so far as it does not take account of the horizontal motion, which must of necessity accompany the vertical elevation of the water. Water being an inelastic fluid, any vertical column of the liquid can only have its length increased by a diminution of its horizontal dimension. It is necessary, therefore, to represent or conceive this horizontal motion as well as the vertical motion.

The horizontal range of motion of the wave is necessarily determined by the volume of the wave. The water which forms the wave is added to the given volume in which the wave is formed, at its posterior extremity, and thence displaces a new volume of water which goes to displace the volume of the wave in the next portion of the channel. Thus the volume of water which occupies the space A′ B′ *b d* before the transit of the wave (see Plate VI. fig. 4), occupies only the length A B *b d* during the wave transit, and it now consists of the rectangle A B *b d*, together with the volume of the wave A C *d*, which volume is equal to the volume A B B′ A′ by which it is replaced; and this happens successively in every point of the fluid. The horizontal range of motion is thus equal to the volume of water employed to form the wave.

While, therefore, the front of the wave is transmitted from A′ to *d*, the water particle A′ is transferred to A. The same particle is also raised and depressed through the height of the wave. These motions in the vertical and horizontal plane are simultaneous. It is required to represent accurately these motions: take $c\,d$ = the height of the wave, A A′ = the range of translation: describe an ellipse whose major axis is the range of translation, and whose semi-minor axis is the height of the wave: describe the wave circle *d* 1, 2, 3, 4, *c*, and having divided as formerly its circumference into equal parts, draw the horizontal ordinates 11, 22, 33, 44, &c., as in fig. 3, and let the curve of versed sines A′ C′ *d* be drawn as in fig. 3, then will the curve A′ 8, 7, 6, C′ 4, 3, 2, 1, *d*, represent the wave curve, the vertical motion only being considered. But at the same time that the particle rises and falls through 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 on the diameter *c d*, and in the curve of versed sines, the particle A′ will advance to A, through A′ 1, 2, 3, 4, 5, 6, 7, 8, 9, A. Thus every point in the curve will have to be advanced forward in the direction of translation in order to represent the actual form of the wave. This is done in fig. 4, and also for a larger wave in fig. 5. While the wave rises to 1, 2, 3, 4, C′, &c., it also advances simultaneously at each point by the quantity A′ 1, A′ 2, A′ 3, A′ 4, A′ 5, A′ 6, A′ 7, &c., and thus the wave A′ C′ *d* becomes transformed in both figures into A C *d*. This curve represents the form of the wave as corrected for the horizontal translation. Thus are reconciled to each other the apparently diverse motions of the particles, by one of which it describes the observed sinuous wave surface, and by the other the semiellipse of its path of translation.

Finally, as the motions of translation are equal and simultaneous throughout all particles situated in the same vertical line, the path of translation of each particle is an ellipse having the same major axis with that of the particle on the surface, but having its minor axis less in proportion to its distance from the surface of the liquid in repose. (See Plate I. fig. 5.)

Hence, when the wave is not large, the amplitude of the particle path or range of translation is 3·1416 times the height of the wave; this quantity gradually diminishes as the height increases, and becomes nearly 2· when the height approaches the limit of equality with the height of the wave. But near this limit it is not capable of accurate observation.

Mechanism of the Wave.—The study of the phenomena of the translation of water particles during the transit of a wave is peculiarly valuable, as affording us the means of correctly conceiving the real nature of wave transmission of the first order; it therefore deserves great attention.

We perceive, in the first place, that the vertical arrangement of the water particles is not deranged by wave transmission; that is, if we conceive the whole fluid in repose to be intersected by transverse vertical planes, thin, and of the specific gravity of water, these plans will retain their parallelism during transmission and will not affect that transmission.

We may therefore accurately conceive the whole volume of water as reposing in rectangular vessels, each of them formed between two successive vertical thin movable planes, and bounded by the two sides and bottom of the channel, and above by the plane of repose. The water in each of these elementary vessels undergoes in successive instances the same change as each of the others preceding it, and therefore we may direct our attention to one individual among them.

Let us study the manner in which wave motion is originally communicated to and through each of these elementary columns of fluid.

For this purpose it may be well to recur to the original mode of wave genesis (Plate I. fig. 5). A vertical generating plane P is inserted in the fluid, and forms one of the vertical boundaries of one of the elementary water columns. $\frac{ab}{\alpha\beta}$ $\frac{bc}{\beta\gamma}$ $\frac{cd}{\gamma\delta}$ $\frac{de}{\delta\epsilon}$ $\frac{ef}{\epsilon\zeta}$ $\frac{fg}{\zeta\theta}$ $\frac{gh}{\eta\theta}$ &c. A moving force is applied to P, and the plane communicates to the water column $\frac{ab}{\alpha\beta}$ that pressure; now this water column is bounded on its anterior surface by a similar vertical plane (of water particles) $\frac{b}{\beta}$ in a state of rest, and the effect of this pressure is two-fold, to raise the water column above the level to the height due to the velocity of P, and to diminish the breadth of the column in proportion to the increase of length. Such is the immediate effect of pressure on the plane P. Let us now consider the second (water) plane $\frac{b}{\beta}$; it has now behind it a column of water pressing it forward with a velocity due to its height above the level of repose: it is therefore pressed forward, *à tergo*, just as the plane P originally was pressed forward, only its moving force is measured by the pressure of the column $\frac{ab}{\alpha\beta}$ with a given height above the plane of repose. In all respects the water column $\frac{bc}{\beta\gamma}$ is now in the condition which in the previous moment we found the column $\frac{ab}{\alpha\beta}$. Let us now return to $\frac{ab}{\alpha\beta}$ which is pressed by the plane P with a pressure not only equal to that which raised it to its former height, but with an accelerating force which raises it still higher, and communicates to it a velocity due to that greater height, and also diminishes its breadth in proportion to the increment in height. This new height in the column $\frac{ab}{\alpha\beta}$ is a new increment of pressure on the vertical water plane $\frac{b}{\beta}$, which in its turn presses the water column $\frac{bc}{\beta\gamma}$ in the same manner, with a pressure due to the new height of the water column $\frac{ab}{\alpha\beta}$, raises its height to that due to this pressure, and gives it a corresponding velocity. The third water column $\frac{cd}{\gamma\delta}$ is now in similar circumstances to those of its predecessor $\frac{bc}{\beta\gamma}$ at the preceding instant of time, and is pressed by the plane $\frac{c}{\gamma}$ with a force due to the height of $\frac{bc}{\beta\gamma}$, and the plane $\frac{c}{\gamma}$ now moves forward, raises the height of $\frac{cd}{\gamma\delta}$, and diminishes proportionally its breadth. The same process continues during the acceleration of the original plane P until it ceases to be further accelerated, and now the whole anterior half of the wave has been generated, and the column $\frac{ab}{\alpha\beta}$ is moving with the velocity due to its elevation above the level, or the height due to the crest of the wave, having passed successively through each of the successive conditions of the columns before it. The force acting on P, *à tergo*, is now to be diminished; the pressure back upon its surface, arising from the height of $\frac{ab}{\alpha\beta}$, tends to retard the motion of P, and as the accelerating force is diminished the retardation increases, the whole action of the column $\frac{ab}{\alpha\beta}$ being continually to retard the plane P; and if the diminution of force take place in the same succession as the original increments, the diminution of the velocity of P will take place in a manner similar to that of its original increase, and it will finally be brought to rest when the column $\frac{ab}{\alpha\beta}$ has regained its level.

The same succession of conditions takes place in the plane which separates any two successive elementary columns; first of all the posterior surface of the plane is pressed by a higher column than itself, tending to increase its height and increased velocity, and having reached the maximum, the anterior surface is thereafter pressed by a water column of greater height than the posterior surface, retarding its velocity, and finally bringing it into a state of rest. Thus the forces and motion of each elementary plane are repetitions of the forces and motions of the original disturbing plane by which the wave was generated.

The power employed in wave genesis is therefore expended in raising to a height, equal to the crest of the wave, each successive water column; each water column, again descending, gives out that measure of power to the next in succession, which it thus raises to its own height. The time employed in raising a given column to this height, and in its descent and communication of its own motion to the next in succession, constitutes the period of a wave, and the number of such columns undergoing different stages of the process at the same time measures the length of a wave.

During the anterior half of the wave the following processes take place. The generating force communicates to the adjacent column, through its posterior bounding plane, a pressure; this pressure moves the posterior plane forward, the water in the column is thereby raised to the height due to the velocity, and the pressure of this water column communicates to the anterior bounding plane also a velocity and a pressure in the same direction; therefore the accelerating force produces a given motion of translation in the whole column a height of column due to that velocity, and an approximation of the anterior forces of the column to each other; these are all the forces and the motions concerned in the matter. The motive power thus stored during the anterior half of the wave is restored in the latter half wave length thus: the column raised to its greatest height presses on both its posterior and anterior surface, on the anterior surface it presses forward the anterior column, tending to sustain its velocity and maintain its height; on the posterior column its pressure tends to oppose the progress and retard the velocity of the fluid in motion, and thus retarding the posterior and accelerating the anterior surface, widens the space between its own bounding planes until it repose once more on the original level.

The Wave a Vehicle of Power.—The wave is thus a receptacle of moving power, of the power required to raise a given volume of water from its place in the channel to its place in the wave, and is ready to transmit that power through any distance along that channel with great velocity, and to replace it at the end of its path. In doing this the motion of the water is simple and easily understood, each column is diminished in horizontal dimension and increased proportionally in vertical dimension, and again suffered to regain its original shape by the action of gravity. There is no transference of individual particles through, between, and amongst one another, so as to produce collisions, or any other motions which impair moving force; the particles simply glide for the moment over each other into a new arrangement, and retire back to their places. Thus the wave resembles that which we may conceive to pass along an elastic column, each slice of which is squeezed into a thinner slice, and restored by its elastic force to its original bulk, only in the water wave the force which restores the force of each water column is gravity, not elasticity.

To conceive accurately of the forces which operate in wave transmission, and of the *modus operandi;* to understand how the primary moving force acts on the column of fluid in repose, how this force is distributed among the particles; to distinguish the relative and absolute motions of the particles and the nature of the transmission of the form, and to understand how the force operates in at once propagating itself and restoring completely to rest those particles which form the vehicle of its transmission, is a study of much interest to the philosopher. To show how under a given form and outline of wave, in a given time, all and each of the individual particles of water obeying every one its own impulse and that of those around it, and subject to the laws of gravity and of the original impulse, shall describe its own path without interfering with another's, and shall unite in the production of an aggregate motion consistent with the continuity of the mass and with the laws of fluid pressure,—this is a problem which belongs to the mathematician, which has hitherto proved too arduous for the human intellect, and which we have thus endeavoured to facilitate and promote by the study of the absolute forms and phenomena of the waves themselves, and by the determination of the actual paths and motions of the individual particles of water.

The Negative Wave of the First Order.—The negative wave is a phenomenon whose place among waves it is somewhat difficult to assign. Its phenomena partake of those of the first order. But in its genesis and propagation it is always attended by a train of following phenomena of the second order.

The genesis of the negative wave of the first order is effected under conditions precisely the reverse of those of the positive wave. A solid body, Q_2 Q_3 (Plate VI. figs. 7, 8), is withdrawn from the water of the reservoir at one extremity, a cavity is created, and this cavity, W_1, is propagated along the surface of the water under a defined figure.

The velocity of the negative wave in a shallow channel is nearly that which is due to the depth calculated from the lowest part of the wave (as in the positive from the highest), but in longer waves it is sensibly less than that velocity. In Plate II. fig. 5 the observations are compared with this formula, from which they exhibit considerable deviations. Table XI. is a collection of negative waves observed in a small rectangular channel, and Table XII. contains others made in a triangular channel, both being made under the same conditions as the positive waves already given.

TABLE XI.

Observations on the Velocity of Negative Waves of the First Order.— In a rectangular channel 12 inches wide.

Col. A is the depth of the fluid reckoned in inches from the lowest point of the wave.

Col. B is the depth of the wave reckoned below the surface of repose.

Col. C is the number of seconds observed while the wave described the space given in column D in feet.

Col. E is the resulting velocity.

Col. F gives the velocities due to the depth, calculated by the formula $c = \sqrt{g\ (h-k)}$.

Col. G are the differences between observation and the formula.

A.	B.	C.	D.	E.	F.	G.
·915	− ·085	9·0	14·62	1·62	1·56	− ·06
·925	− ·075	9·5	14·62	1·53	1·57	+ ·04
·93	− ·07	16·5	21·08	1·27	1·58	+ ·31
·935	− ·065	12·0	20·0	1·66′	1·58	− ·08
·96	− ·04	14·5	20·0	1·38	1·60	+ ·22
·965	− ·035	15·0	21·08	1·40	1·60	+ ·20
·97	− ·03	14·0	20·5	1·46	1·61	+ ·15
1·0					1·63	
2·0					2·31	
3·0					2·83	
3·3	− ·8	5·5	14·62	2·65	2·97	+ ·32
3·4	− ·7	6·0	14·62	2·43	3·02	+ ·59
3·495	− ·605	8·0	21·08	2·63	3·08	+ ·45
3·603	− ·497	13·5	41·08	3·04	3·10	+ ·06
3·71	− ·39	6·5	20·0	3·07	3·15	+ ·08
3·745	− ·355	7·0	20·0	2·85	3·16	+ ·31
3·77	− ·33	10·83′	33·3′	3·07	3·18	+ ·11
4·0					3·27	
4·365	− ·735	4·25	14·62	3·44	4·42	− ·02
4·575	− ·525	6·0	20·0	3·33′	3·50	+ ·17
4·6	− ·5	6·25	21·08	3·37	3·51	+ ·14
4·625	− ·475	7·5	20·0	2·66	3·52	+ ·86
4·75	− ·35	5·25	20·0	3·81	3·57	− ·24
5·0					3·66	
6·0					4·01	
7·0					4·33	
						+ 4·01
						− 0·40
						+ 3·61
					Mean.	+ 0·19

TABLE XII.

*Observations on the Velocity of Negative Waves of the First Order.—
In a triangular channel with sides sloping at 45°.*

Cols. A, B, C, D, E, F and G, as in the preceding table.

Col. H is the ratio of defective velocity on the whole.

Col. F″ is taken, not from the formula like F, but from observed positive waves in the same channel of the same height.

Col. G″ contains the differences between F″ and E.

A.	B.	C.	D.	E.	F.	G.	H.	F″.	G″.
8·7	−0·7	29·8	100·	3·35	3·41	+ ·06	·0179	3·27	− ·08
8·8	−0·6	92·4	315·5	3·41	3·43	+ ·02	·0058	3·31	− ·10
8·9	−0·5	62·8	215·5	3·43	4·45	+ ·02	·0058	3·35	− ·08
9·0					3·47				
10·0					3·66				
11·0					3·84				
11·6	−0·9	29·2	100·	3·41	3·94	+ ·53	·1554	3·78	+ ·37
12·0					4·01				
13·0					4·17				
14·0					4·33				
15·0					4·48				
16·0					4·63				
16·8	−1·7	22·2	100·	4·50	4·74	+ ·24	·0533	4·50	·00
17·0	−1·5	22·0	100·	4·54	4·77	+ ·23	·0506	4·55	+ ·01
17·4	−1·1	21·6	100·	4·62	4·83	+ ·21	·0454	4·67	+ ·05
18·0	−0·5	21·6	100·	4·62	4·91	+ ·29	·0627	4·84	+ ·22
19·0					5·04				
20·0					5·18				
21·0					5·30				
22·0					5·43				
23·0					5·55				
24·0					5·67				
24·5	−1·5	19·0	100·	5·26	5·73	+ ·47	·0893	5·55	+ ·29
24·7	−1·3	18·9	100·	5·29	5·75	+ ·46	·0869	5·60	+ ·31
24·8	−1·2	18·6	100·	5·38	5·76	+ ·38	·0706	5·62	+ ·24
25·0	−1·0	18·6	100·	5·38	5·78	+ ·40	·0743	5·67	+ ·29
						+3·31	·7180		+1·78
					Mean.	+0·275	·0598	Mean.	− ·26
									+1·52
								Mean	+ ·126

The horizontal translation of water particles in the negative wave presents considerable resemblance to the corresponding phenomenon in the positive wave. All the particles of water in a given vertical plane move simultaneously with equal velocities backwards in the opposite direction to the transmission, and repose in their new planes, at the end of the translation; with this modification, however, that this state of repose is much disturbed near the surface by those secondary waves which follow the negative wave, but which do not sensibly agitate the particles considerably removed from the surface. (See Plate VI. fig. 9.) The path is the ellipse of the positive wave inverted.

The following measures may be useful. In a rectangular channel 4 inches deep in repose and 8 inches wide, a volume of 72 cubic inches is withdrawn; the depth of the negative wave below the plane of repose is $\frac{5}{8}$ths of an inch deep, the translation throughout the lower half-depth is $2\frac{1}{4}$ inches, and diminishes from the half-depths upwards, settling finally at the surface at $1\frac{3}{4}$ inch from the original position of the superficial particle.

The form of surface of the anterior half of the negative wave resembles closely the posterior half of a positive wave of equal depth, but the posterior half of the negative wave passes off into the anterior form of a secondary wave which follows it.

After translation the superficial particles continue to oscillate, as shown in Plate VI. figs. 9, 10, in the manner hereafter to be described, as a phenomenon of the train of secondary waves.

The characteristics of this species of wave of the first order are:—

(1.) That it is negative or wholly below the level of repose.

(2.) That it is a wave of translation, the direction of which is opposite to the direction of transmission.

(3.) That its anterior form is that of the positive wave reversed.

(4.) That the path of translation is nearly that of the positive wave reversed.

(5.) That its velocity is, in considerable depths, sensibly less than that due by gravity to half the depth reckoned from the lowest point, or the velocity of a positive wave being the same total height.

(6.) That it is not solitary, but always carries a train of secondary waves.

It is important to notice that the positive and negative waves do not stand to each other in the relation of companion phenomena. They cannot be considered in any case as the positive and negative portions of the same phenomena, for the following reasons:—

(1.) If an attempt be made to generate or propagate them in such manner that the one shall be companion to the other, they will not continue together, but immediately and spontaneously separate.

(2.) If a positive wave be generated in a given channel and a negative wave behind it, the positive wave moving with the greater velocity, rapidly separates itself from the other, leaving it far behind.

(3.) If a positive wave be generated and transmitted behind a negative wave, it will overtake and pass it.

(4.) Waves of the secondary class which consist of companion halves, one part positive and the other negative, have this peculiarity, that the positive and negative parts may be transmitted across and over each other without preventing in any way their permanence or their continued propagation. It is not so with the positive and negative waves of the first order.

(5.) If a positive and negative wave of equal volume meet in opposite directions, they neutralise each other and both cease to exist.

(6.) If a positive wave overtake a negative wave of equal volume, they also neutralise each other and cease to exist.

(7.) If either be larger, the remainder is propagated as a wave of the larger class.

(8.) Thus it is nowhere to be observed that the positive and negative wave co-exist as companion phenomena.

These observations are of importance for this reason, that it has been supposed by a distinguished philosopher that the positive and the negative wave might be corresponding halves of some given or supposed wave.

On some Conditions which affect the Phenomena of the Wave of the First Order.—It has not appeared in any observations I have been able to make on the subject, that the wave of the first order retains the stamp of the many peculiarities that may be conceived to affect its origin. In this respect it is apparently different from the waves of sound or of colour, which bear to the ear and the eye distinct indications of many peculiarities of their original exciting cause, and thus enable us to judge of the character of the distant cause which emitted the sound or sent forth the coloured ray. It is not possible always to form an accurate judgment from the phenomena of the wave of the first order, of the nature of the disturbing cause, except in peculiar and small number of cases.

I have not found that waves generated by impulse by a fluid column of given and very various dimension, by immersion of a solid body of given figure, by motion in given velocity or in different directions; I have not found in the wave obtained by any of the many means any peculiarity, any variation either of form or velocity, indicating the peculiarity of the original. In one respect therefore the wave of translation resembles the sound wave; that all waves travel with the velocity due to half the depth, whatever be the nature of their source.

In one respect alone does the origin of the wave affect its history. Its volume depends on the quantity of power employed in its genesis, and on the distance through which it has travelled. A great and a little wave at equal distances from the source of disturbance, arise from great or little causes, but it is impossible to distinguish between a small wave which has travelled a short distance, and one which, originally high, has traversed a long space.

This, however, does not apply to compound waves of the first order, hereafter to be examined.

Form of Channel.—Its Effect on the Wave of Translation.—The conditions which affect the phenomena of the wave of translation are therefore to be looked for in its actual circumstances at the time of observation rather than in its history. The form and magnitude of the channel are among the most important of these circumstances. Thus a change in depth of channel immediately becomes indicated to the eye of the observer by the retardation of the wave, which begins to move with the same velocity as if the channel were everywhere of the diminished depth, that is, with the velocity due to the depth. Thus in a rectangular channel $4\frac{1}{2}$ feet deep, the wave moves with a velocity of 12 feet per second, and if the channel become shallower, so as to have only 2 feet depth, the change of depth is indicated by the velocity of the wave, which is observed now to move only with the velocity of 8 feet per second; but if the channel again change and become 8 feet deep, the wave indicates the change by suddenly changing to a velocity of 16 feet per second.

Length of Wave an Index of Depth.—In like manner, a wave which in water 4 feet deep is about 8 yards long, shortens on coming to a depth of 2 feet to a length of 4 yards, and extends itself to 16 yards long on getting into a depth of 8 feet. This extension of length is attended with a diminution of height, and the diminution of length with an increase of height of the wave, so that the change of length and height attend and indicate changes of depth.

In a rectangular channel whose depth gradually slopes until it becomes nothing, like the beach of a sea, these phenomena are very distinctly visible; the wave is first retarded by the diminution of depth, shortens and increases in height, and finally breaks when its height approaches to equality with the depth of the water. The limit of height of a wave of the first order is therefore a height above the bottom of the channel equal to double the depth of the water in repose. If we reckon the velocity of transmission as that due to half the total depth, and the velocity of translation as that due to the height of the wave, it is manifest that when the height is equal to the depth these two are equal, but that if the height were greater than this, the velocity of individual particles at the crest of the wave would exceed the velocity of the wave form; here accordingly the wave ceases, the particles in the ridge of the wave pass forward out of the wave, fall over, and the wave becomes a surge or broken foam, a disintegrated heap of water particles, having lost all continuity.

In like manner does the gradual narrowing of the channel affect the form and velocity of the wave, but its effects are by no means so striking as where the depth is diminished. The narrowing of the channel increases the height of the wave, and the effect of this is most apparent when the height is considerable in proportion to the depth; the velocity of the wave increases in proportion as the increase of height of the wave increases the total depth; but with this increase of depth, the length of the wave also increases rapidly, and it does not break so early as in the case of the shallowing of the water. Its phenomena are only visibly affected to the extent in which a change of depth is produced in the channel, by the volume of water added to the channel taking the velocity and form peculiar to that increased depth.

TABLE XIII.

Observed Heights of a Wave in Channel of variable Breadth.—Depth 4 inches.

	A. Breadth 12 in. Height of wave.	B. Breadth 6 in. Height of wave.	C. Breadth 3 in. Height of wave.
	in.	in.	in.
I.	2·0	2·4	3·3
II.	2·0	2·4	3·6
III.	2·0	2·55	3·3
IV.	1·5	2·5	3·5
V.	1·5	2·35	3·25
VI.	1·25	2·0	2·5
VII.	1·0	1·3	2·0
VIII.	0·25	0·3	0·4

These numbers appear to indicate that the increase of height does not widely differ from the hypothesis, that the height of a given wave in a channel of variable width is inversely as the square root of the breadth.

Thus, the inverse square roots of the breadths are as 1·73, 2·45, and 3·47, and the mean heights of the first five experiments are 1·8, 2·45, 3·39.

In the first five experiments the velocity observed was 4·25 feet per second. The velocity due by gravity to half the total depth $4 + 2·45$ inches is 4·15 feet per second; and as the range of the wave was only 17 feet, and the time was only observed to half-seconds, these numbers coincide well enough to bear the conclusion that the velocity does not considerably differ from that due to the wave of the same mean height in a parallel channel of the same depth.

Table XIV.

Observations in a Channel of variable Depth.—Diminution of depth from 4 inches to 0· in a length of 17 feet.

	A. Height of wave in a depth of 4 in.	B. Height of wave breaking in depth (C).	C. Depth of water where wave (B) broke.	D. Time of traversing 17 ft.	E. Velocity in feet per sec.
	in.	in.	in.	s.	
I. ...	4·0*	4·0*	4·0	5·5	3·09
II. ...	3·7*	3·7*	3·7	5·5	3·09
III. ...	3·4*	3·4*	3·4	5·5	3·09
IV. ...	2·5	2·7	2·7	5·5	3·09
V. ...	2·0	2·4	2·4	5·5	3·09
VI. ...	1·8	2·2	2·2	5·5	3·09
VII. ...	1·5	2·0	2·1	...	...
VIII. ...	1·3	1·9	1·9	...	...
IX. ...	1·25	1·9	1·9	...	...
X. ...	1·2	1·7	1·7	...	...
XI. ...	1·1	1·4	1·4	6·0	2·83
XII. ...	1·0	1·2	1·2	..	...
XIII. ...	0·8	0·8	1·1	6·5	2·6
XIV. ...	0·5	0·7	0·9	7·0	2·4
XV. ...	0·2*	0·2*	0·2	7·5	2·0

Hence we find that the numbers representing depths in column C may be regarded as the limits of those in column B, that the depth of the fluid below the level of repose is equal to the greatest height which a wave can attain at that point, and at that height the wave breaks.

The time occupied by the largest class of wave is 5·5 seconds, and the corresponding *mean* velocity is 3·09 feet per second; this is the velocity due to a depth of 3·6 inches, but the depth total at the one end of the channel is nearly double this quantity, diminishing to 0 at the end. The time in which the wave in a shelving channel passes along the whole length, is therefore nearly equal to the time in which a wave would travel the same distance if the channel were uniformly of a depth equal to the mean depth of the channel, reckoning in both cases from the top of the wave. In these cases the height of the wave is large. Let us take a small height of wave as Ex. XIV.; there we have also in this case the mean depth reckoned from the top of the wave = 2·2, the velocity in a channel of that uniform depth = 2·4, and the time 7ˢ·08. These experiments are sufficiently accurately

* These numbers are interpolated; the numbers in column D are waves not observed on the identical waves in the first three columns, but are others of nearly equal heights, in identical conditions.

represented if we take for the velocity of the wave in the sloping channel that of a wave in a channel having a uniform depth equal to the mean depth of the channel, reckoned as usual from the top of the wave.

If therefore we are to calculate the time in which a wave will traverse a given distance q, to the limit of the standing water-line, after it has begun to break on a sloping beach, we have, the height at breaking being $h =$, the standing depth of the water at the break-point,

$$t = \frac{q}{\sqrt{g(h + k)}} \text{ and } v = \sqrt{g(h + k)}.$$

Ex. A wave 3 feet high breaking in water 3 feet deep, on a sloping shore at a distance of 60 feet from the edge of the water, would traverse that space in about 6 seconds, for

$$t = \frac{60}{32 \cdot 3 \sqrt{}} = \frac{60}{9 \cdot 82} = 6 \text{ seconds nearly.}$$

By repeated observations I have ascertained that waves break whenever their height above the level of repose becomes equal very nearly to the depth of the water.

The gradual retardation of the velocity of waves breaking on a sloping beach, as they come into shallower water, is rendered manifest in the closer approximation of the waves to each other as they come near the margin of the water. Vide *et seq.*

It may be observed also that the height of the wave does increase, but very slowly (before breaking), as the depth diminishes; thus in VII., a height of 1·8 in a depth of 4 inches becomes 2·2 in 2 inches depth, and in XII. a height of 1 inch in a depth of 4 inches becomes a depth of 1·2 inch only 1·2 inch high. The increase of height is therefore very much slower than the inverse ratio of the depth, or than the inverse ratio of the square of the depth.

Form of Tranverse Section of Channel.—We have seen that in a given rectangular channel, the volume of the wave, its height and the depth being given, no peculiarity of origin or other condition sensibly affects its actual phenomena. But it becomes of importance to know whether the form of a given channel, its volume being given, will affect the phenomena of the wave of the first order; for example, whether in a channel, which is semicircular on the bottom, or triangular, but holding a given quantity of water, the wave would be affected by the form of the channel, the volume or cross section remaining unchanged.

Considering this question *à priori*, we might form various anticipations. We might expect in a channel in which the depth of transverse section varies, that as its depth is greatest at one point, suppose the middle, and less at the sides, the wave might move with the velocity due to the middle or greatest depth; or we might expect that it would move with the velocity simply due to the mean depth, that is, with the same velocity as in a rectangular channel of a depth equal to the mean depth of the channel; or we might expect that each portion of the wave would move with a velocity due to the depth of that part of the channel immediately below each part of the wave, and so each part passing forward with a velocity of its own, have a series of waves, each propagating itself with an independent velocity, and speedily becoming diffused, and so a continued propagation of a wave in such circumstances would become *impossible* from disintegration; and instead of a single large wave we should have a great many little ones. Or, finally, we might have a perfect wave moving with a velocity, the mean of the velocities which each of these elementary waves might be supposed to possess.

I soon found that the propagation of a single wave, *i.e.*, one of which all the parts should have a given common velocity, was *possible* in a channel whose depth at different breadths is variable; that the wave does not necessarily become disintegrated; that its parts do not move with the different velocities due to the different depths of the different parts of the channel, but that the entire wave does (with certain limits) move with such velocity as if propagated in a channel of a rectangular form, but of a less depth than the greatest depth of the channel of variable channel.

It became necessary, therefore, to determine the depth of a rectangular channel equivalent to the depth of a channel of variable transverse section; to determine, for example, in a channel of triangular section ▽, the depth of rectangular channel in which a wave would be propagated with equal velocity. In this case the simple arithmetical mean depth of the channel is *half of the depth in the middle.* But on the other hand, if we calculate the velocity due to each point of variable depth, and take the mean of these velocities, we shall find a mean velocity such as would be due to a wave in a rectangular channel *two-thirds of the greatest depth.*

In the first series of experiments I made on this subject, I conceived that the results coincided sufficiently well with the latter supposition; but they were on so small a scale, that the errors of observation exceeded in amount the differences between the quantities to be determined, and the results did not establish either. Mr. Kelland arrived at the opposite conclusion, his theoretical investigations indicating the former result. I examined the matter afresh, and after an extensive series of experiments, have established beyond all question the fact, that the velocity in a triangular channel is that due by gravity to one-fourth of the maximum depth. Although therefore the absolute velocity assigned by Mr. Kelland's investigations deviates widely from the true velocity, yet he has assigned the true relation between the velocities in the triangular and the rectangular channel; and if therefore we take the absolute velocity which I have determined for the rectangular channel, and deduce from it the relative velocity which Mr. Kelland has assigned to the triangular form, we obtain a number which is the true velocity of the wave in a $\triangledown^r$ channel.

TABLE XV.

Observations on the Wave of the First Order in triangular Channels.

The sides of the channels are planes, and slope at an angle with the horizon $= 45°$.

Col. A is the observed depth of the channel in the middle, reckoned from the crest of the wave.

Col. B is the height of the wave taken as the mean between the observations at the beginning and end of the experiment.

Col. C is the observed time in seconds occupied by the wave in describing the distance in column D.

Col. D is the space in feet described by the wave during each observation.

Col. E is the velocity resulting from these observations.

Col. F is the velocity due by gravity to $\frac{1}{4}$ of the depth of the fluid, $v = \sqrt{\frac{1}{2} g (h + k)}$.

Col. G is the velocity due by gravity to $\frac{2}{3}$ of the depth of the fluid, $v = \sqrt{\frac{2}{3} g (h + k)}$.

Cols. H and K show the difference between Cols. F and G and the observations, and the result in favour of F.

A.	B.	C.	D.	E.	F.	G.	H.	K.
in.	in.							
4·15	0·15	36·5	80·0	2·19	2·35	2·72	+ ·16	+ ·53
4·23	0·22	33·0	80·0	2·42	2·38	2·75	− ·04	+ ·33
4·32	0·31	31·0	75·5	2·43	2·40	2·78	− ·03	+ ·35
4·38	0·37	47·0	115·5	2·46	2·42	2·79	− ·04	+ ·33
4·71	0·70	13·5	35·5	2·62	2·51	2·90	− ·11	+ ·28
4·81	0·80	29·5	75·5	2·57	2·54	2·93	− ·03	+ ·36
4·86	0·85	14·0	35·5	2·53	2·55	2·95	+ ·02	+ ·42
5·29	0·18	31·0	80·0	2·58	2·66	3·07	+ ·08	+ ·49
5·44	0·33	45·5	120·0	2·63	2·70	3·11	+ ·07	+ ·48
5·55	0·44	58·0	160·0	2·75	2·72	3·15	− ·03	+ ·40
5·59	0·48	30·0	80·0	2·66	2·73	3·16	+ ·07	+ ·50
5·99	0·88	12·0	35·5	2·95	2·83	3·27	− ·12	+ ·32
6·01	0·90	24·5	71·0	2·89	2·84	3·29	− ·05	+ ·40
6·18	0·14	28·0	80·0	2·85	2·87	3·32	+ ·02	+ ·47
6·26	0·21	55·5	160·0	2·88	2·89	3·34	+ ·01	+ ·46
6·38	0·34	14·0	40·0	2·85	2·92	3·37	+ ·07	+ ·52
6·44	1·33	12·0	35·5	2·95	2·93	3·39	− ·02	+ ·44
6·52	0·48	26·5	80·0	3·02	2·95	3·41	− ·07	+ ·39
6·78	0·74	35·0	111·0	3·17	3·01	3·48	− ·16	+ ·31
7·10	0·60	26·5	80·0	3·02	3·08	3·56	+ ·06	+ ·54
7·12	0·08	39·5	120·0	3·03	3·09	3·56	+ ·06	+ ·53
7·15	0·11	78·5	240·0	3·05	3·09	3·57	+ ·04	+ ·52
7·16	0·12	52·5	160·0	3·04	3·10	3·58	+ ·06	+ ·54
7·21	0·17	26·5	80·0	3·02	3·11	3·59	+ ·09	+ ·57
7·36	0·32	26·5	80·0	3·02	3·14	3·62	+ ·12	+ ·60
7·51	0·47	25·0	80·0	3·20	3·18	3·66	− ·02	+ ·46
7·53	0·47	24·0	80·0	3·33	3·17	3·67	− ·16	+ ·34
10·0	0·75	55·4	215·5	3·89	3·66	4·23	− ·23	+ ·34
10·5	1·1	41·94	166·0	3·95	3·75	4·33	− ·20	+ ·38
11·0	1·44	31·2	123·1	3·94	3·84	4·43	− ·10	+ ·49
14·5	2·0	48·36	215·5	4·45	4·41	5·09	− ·04	+ ·64
15·0	2·58	26·46	119·25	4·50	4·48	5·18	− ·02	+ ·68
15·5	3·1	22·2	100·0	4·50	4·56	5·26	+ ·06	+ ·76
19·0	0·35	19·8	100·0	5·06	5·04	5·83	− ·02	+ ·77
19·5	0·87	19·5	100·0	5·13	5·11	5·90	− ·02	+ ·77
20·0	1·35	25·66	138·5	5·40	5·18	5·98	− ·22	+ ·58
20·5	1·85	28·8	157·75	5·48	5·24	6·05	− ·24	+ ·57
21·0	2·36	24·93	138·5	5·55	5·30	6·13	− ·25	+ ·58
21·5	2·8	17·8	100·0	5·61	5·36	6·20	− ·25	+ ·59
26·0	1·5	35·8	215·5	6·02	5·90	6·82	− ·12	+ ·80
26·5	1·95	22·46	138·5	6·16	5·96	6·88	− ·20	+ ·72
27·0	2·12	20·7	128·87	6·22	6·01	6·95	− ·21	+ ·73
27·5	2·4	21·73	138·5	6·37	6·07	7·01	− ·30	+ ·64
28·0	3·12	20·45	128·75	6·29	6·13	7·07	− ·16	+ ·78
28·5	3·03	15·93	100·0	6·27	6·18	7·14	− ·09	+ ·87
29·0	3·02	15·8	100·0	6·33	6·23	7·20	− ·10	+ ·87
29·5	2·5	15·68	100·0	6·37	6·29	7·26	− ·08	+ ·89
30·0	2·77	15·6	100·0	6·41	6·34	7·32	− ·07	+ ·91
30·5	2·25	15·6	100·0	6·41	6·39	7·38	− ·02	+ ·97
31·0	2·5	15·8	100·0	6·33	6·44	7·44	+ ·11	+ 1·11
31·5	3·0	15·26	100·0	6·55	6·50	7·50	− ·05	+ ·95
							−2·77	+29·27

No great number of experiments has been made on channels of other forms of variable depth, such as have been made coinciding with those in the triangular channel, so far as to show that we may take the simple arithmetical mean depth as the depth of the rectangular channel of a wave of equal velocity, and so in general reckon the mean depth as

$$h = \frac{1}{x}\int y dx,$$

or $$v = \left(\frac{g}{x}\int y dx\right)^{\frac{1}{2}}.$$

The form of transverse section does not therefore affect the velocity of the wave otherwise than as it becomes necessary to use the mean depth as the argument in calculating it, and not the maximum depth.

The Form of Channel affects the Form of the Wave as well as its Velocity.—When the channel is very broad the wave ceases to have a velocity, it loses unity of character, and each part of it moves along the channel independent of the velocity of the other, and with the velocity due to the local depth of the channel. Where the water is shallow the wave becomes sensibly higher and shorter, and when the difference of depth is not considerable, the wave is found to increase in height so as to give in the shallow part a velocity equal to that in the narrow part. When the channel is narrow in proportion to its depth, this unity of propagation exists without sensible difference of velocity towards the side, and without very great difference in height at the sides. In a channel of the form of a right-angled and isosceles triangle, with the hypothenuse upwards and horizontal, it is visible to the eye that the wave is somewhat longer and lower in the middle, but higher and shorter at the sides, but that it retains most perfect unity of form and velocity, and moves along unbroken with the velocity due to the mean depth. The same figure with the angle at the bottom increased so that each side has a slope of one in four, still contains a single wave propagated with a single velocity, being that due to half the depth, but breaks at the shallow side, becoming disintegrated in form though not in velocity.

In a channel 12 inches wide, 5 inches deep on one side, and 1 inch deep on the other, the following observations were made:—

Height of the Wave.

Deep side. in.	Shallow side. in.
2·00	2·50
1·50	2·50
1·20	2·00
0·75	1·20
0·75	1·20
0·75	1·00
0·50	1·00
0·25	0·50
0·25	0·40
0·25	0·40

On the Incidence and Reflection of the Wave of the First Order.— When a wave of the first order encounters a solid plane at right angles to the direction of its propagation, it is wholly reflected, and is thrown back in the opposite direction with a velocity equal to that in which it was moving before impact, remaining in every respect unchanged, excepting in direction of motion. This process may be repeated any number of times without affecting any of the wave phenomena excepting the direction of motion.

When the angle which the ridge of the accident wave makes with the solid plane is small, that is, when the direction of propagation does not deviate much from the perpendicular to the plane, the wave undergoes total reflexion, and the angles of reflexion and of incidence are equal, as in the case of light.

When the deviation of the direction of propagation from the perpendicular is considerable, the reflexion ceases to be total. At 45° the reflected wave is sensibly less than the incident wave.

When the ridge of the wave is incident at about 60° from the plane surface, and the direction of the ridge only diverges about 30° from a perpendicular to the plane, reflexion ceases to be possible. A remarkable phenomenon is exhibited which I may be allowed to designate the *Lateral Accumulation* and *Non-Reflexion* of the wave. It is to be understood by considering the effect of supposed reflexion; this would be to double over upon itself a part of the wave moving in nearly the same direction; the motions of translation of the particles being compounded will give a resultant at right angles to the plane, and will also give a wave of greater magnitude and a translation of greater velocity. By these means accumulation of volume and advancement of the ridge in the vicinity of the obstacle take place; as represented in the diagram.

These phenomena are accurately represented in Plate VII., as observed in a large shallow reservoir of water.

On the Lateral Diffusion and the Lateral Accumulation of the Wave of the First Order.—When a wave of the first order has been generated in a narrow channel, and is propagated into a wider one, it becomes of some importance to know whether and how this wave will affect the surface of the larger basin into which it is admitted. It is known that common surface waves of the second order diffuse themselves equably in concentric circles round the point of disturbance. How is the great primary wave diffused?

TABLE XVI.

Observations on the Lateral Diffusion of the Wave of the First Order, generated in a narrow Channel and transmitted into a wide Reservoir.

The apparatus employed for this purpose is exhibited in Plate VIII. figs. 1 and 2. T was a tank 20 feet square, filled to the depth of 4 inches; the chamber C, fig. 2, was 12 inches square, in which the wave was generated by impulse for the first five experiments, in all subsequent to which C was enlarged in width to 2 feet, as shown in fig. 1. The line marked A, figs. 1 and 2, was a wooden bar, in which were inserted at intervals of 6 inches, sharp pieces of pencil, projecting downwards to the surface of the water; the numbers of which, reckoning from the side of the tank outwards, are contained in the first vertical column of numerals, the Roman numerals in this table denoting the number of the experiment. The bar being placed parallel to the side of the tank at C, and distant from it 12 feet, consequently distant 9 feet from the mouth of the channel, whose length is 3 feet; the distance from its under edge to the surface of the still water was carefully measured, and when the wave had passed, and before its reflexion, the bar was removed, the distances from its under edge to the highest marks on the pencils were put down in column A of the table, and the absolute height of the wave itself, obtained by subtracting these figures from the statical level, was put down in column B.

In the diagrams, Plate VIII., the waves are laid down from the line A A, and at horizontal intervals of one-tenth of an inch, corresponding to the relative positions of the points at which they were observed. In figs. 1 and 2, an approximate mean is given of the waves generated in the large and small channels, each line at the bar A indicating a height of one-tenth part of an inch.

	I.		II.		III.		IV.		V.		VI.	
	A.	B.	A.	B.	A.	B.	A.	B.	A.	B.	A.	B.
1	2·3	·375	2·15	·525	2·05	·625	2·1	·575	·8	·7	·3	1·2
2	2·4	·275	2·175	·5			2·15	·525	·85	·65	·4	1·1
3	2·45	·225	2·225	·45	2·15	·525	2·2	·475	·95	·575	·5	1·0
4	2·45	·225	2·2	·475	2·15	·525	2·15	·525	·8	·725	·4	1·225
5	2·45	·225	2·225	·45	2·15	·525	2·15	·525	·85	·675	·4	1·225
6	2·45	·225	2·3	·375	2·2	·475	2·2	·475	·925	·612	·45	1·075
7	2·475	·2	2·275	·4	2·2	·475	2·25	·425	·925	·625	·5	1·035
8	2·5	·175	2·35	·325	2·25	·425	2·275	·4	1·25	·5	·625	·925
9	2·55	·125	2·4	·275	2·3	·375	2·35	·325	1·05	·5	·625	·925
10	2·55	·125	2·35	·325	2·3	·375	2·3	·375	1·0	·55	·65	·9
11	2·6	·075	2·4	·275	2·3	·375	2·325	·35	1·1	·45	·7	·85
12	2·6	·075	2·425	·25	2·35	·325	2·375	·3	1·05	·512	·825	·75
13	2·6	·075	2·425	·25	2·35	·325	2·35	·325	1·075	·487	·8	·775
14	2·6	·075	2·45	·225	2·375	·3	2·4	·275	1·1	·487	·8	·775
15	2·65	·025	2·45	·225	2·375	·3	2·4	·275	1·1	·475	·9	·675
16	2·65	·025	2·45	·225	2·4	·275	2·4	·275	1·1	·475	·85	·735
17	2·65	·025	2·45	·225	2·4	·275	2·4	·275			·85	·75

This table shows in column B, how the height of the wave diminishes as it spreads out from the line of original direction in which it was generated. Lateral diffusion therefore takes place, but with a great diminution of height of the wave.

This phenomenon is of importance in reference especially to the law of diffusion of the tides, in such situations as where they enter the German Sea through the English Channel, and the Irish Sea through St. George's Channel. It enables us to account for the great inequality of tides in the same locality. It likewise furnishes an analogy by which we may explain some of the hitherto anomalous phenomena of sound.

Axis of Maximum Displacement of the Wave of the First Order.—That a wave of the first order, on entering a large sheet of water, does not diffuse itself equally in all directions around the place of disturbance (as do the waves of the second order produced by a stone dropped in a placid lake), but that there is in one direction *an axis* along which it maintains the greatest height, has the widest range of translation, and travels with greatest velocity, viz., in the direction of the original propagation as it emerged from the generating reservoir, is a phenomenon which I have further confirmed by a number of experiments. This phenomenon is of importance, especially if we take the wave of the first order, the same (as I think I have established) as type of the tide wave of the sea and of the sound wave of the atmosphere. I determined this in the simplest way. I filled a reservoir which has a smooth flat bottom and perpendicular sides some 20 feet square, to a depth of 4 inches with water. In a small generating reservoir only a foot wide, I generated a wave of the first order. A circle was drawn on the bottom of the large basin, and of course visible through the water, having its centre at the place of disturbance, and divided into arcs of 30°, 45°, 60° and 90°, on which observers were placed, and the heights of the same wave, as observed at the points, is given in the accompanying table.

Table XVII.

Observations on the Diffusion of the Wave of the First Order round an Axis of original Transmission.

The observations were made upon the wave at various points in circles of 9 and 15 feet radius, described from the outer extremity of the side of the channel C, as shown in Plate VIII. fig. 3. The depth of the water when at rest was taken at the various points, and these being subtracted from the absolute height to which the wave attained in its transit, gave the amounts which are contained in the lower part of the table, the absolute heights from which these are deduced being given immediately above in columns marked thus, A, B, C, D, E, while the deducted heights are distinguished thus, A′, B′, C′, D′, E′. Experiments VII. to XV. were made in the 9 feet circle, and the remainder in that of 15 feet radius. It will be observed that in the latter set there are two columns which are headed zero, but it must be remembered that the one in brackets contains observations which were made at the 9 feet distance along the axis and the remainder on the outer circle.

Fig. 3 contains the approximate ratio of the height of the wave at different points in the circumference of the circles expressed by lines concentric to the circles, each of which denotes the tenth part of an inch.

The observations are laid down accurately in the diagrams, where the lines A B and C D represent the circumference of the quadrants of the observed circles. Upon these lines the true heights of the wave are measured upwards at their respective points of observation, and a curve drawn through these, representing the mean of the wave's height. From these and from a numerical discussion of the observations, it appears that the height of the wave at 0° being 1, its height at the remaining points will be $\frac{1}{2}$, $\frac{2}{5}$, $\frac{1}{3}$, and $\frac{1}{10}$, or taking integral numbers to express the ratio, it will stand thus, 30, 15, 12, 10, 3. And from a discussion of the whole of the experiments it is found that the height of the wave is inversely as the distance from the centre.

Fig. 4 shows the appearance of the wave upon which these observations were made.

	A.	B.	C.	D.	E.		G.	H.	K.	L.	M.
	0°.	30°.	45°.	60°.	90°.		(0°.)	0°.	30°.	60°.	90°.
VII.	4·5		4·15		4·05	XVI.		4·25	4·3		4·4
VIII.	4·625		4·35		4·05	XVII.		4·125	4·5		
IX.	4·875		4·4		4·05	XVIII.		4·25	4·5		
X.	4·5	4·35			4·05	XIX.		4·25	4·4		
XI.	4·325	4·3			4·05	XX.		4·25	4·3		
XII.	4·5	4·3			4·05	XXI.		4·25		4·3	
XIII.	4·5	4·2				XXII.		4·375		4·3	
XIV.	4·75			4·3	4·05	XXIII.		4·1		4·3	
XV.	4·5			4·25	4·05	XXIV.		4·1		4·25	
	A′.	B′.	C′.	D′.	E′.		G′.	H′.	K′.	L′.	M′.
VII.	1·0		·3		·1	XVI.		·75	·3		·1
VIII.	1·125		·5		·1	XVII.	·7	·625	·5		
IX.	1·375		·55		·1	XVIII.		·75	·5		
X.	1·0	·55			·1	XIX.		·75	·4		
XI.	·825	·5			·1	XX.	1·3	·75	·3		
XII.	1·0	·5			·1	XXI.	1·5	·75		·25	
XIII.	1·0	·4				XXII.		·875		·25	
XIV.	1·25			·4	·1	XXIII.	1·1	·6		·25	
XV.	1·0			·35	·1	XXIV.	1·15	·6		·2	

Thus it was determined that along the axis of maximum intensity, the height of the wave there being the greatest, there was a corresponding acceleration of the wave motion. On each side of this axis the magnitude of the wave diminishes rapidly, being at 30° diminished to $\frac{1}{2}$, and at 60° to $\frac{1}{3}$ of its height along the axis, and as this diminution was attended with a corresponding retardation of propagation, so the ridge of the wave became somewhat elliptical, having for its major axis the axis of maximum intensity of the wave. At right angles to the principal axis of propagation the wave is scarcely sensible, a height of one-tenth part of that in the axis being the greatest that was observed; and that indeed was, in the circumstances of observation, scarcely sensible.

Concluding Remarks and Application.—There are several great applications of our knowledge of waves of the first order, which give value to that knowledge beyond that which belongs to truth for its own sake. The phenomena of the wave of translation are so beautiful and regular, that as a study of nature it possesses a high interest. The velocity of the wave is one of the great constants of nature, and is to the phenomena of fluids what the pendulum is to solids, a connecting link between time and force; as a phenomenon of hydrodynamics, it furnishes one of the most elegant and interesting exercises in the calculus of the wave mathematics.

But besides its importance in these aspects, there are others in which it is capable of being regarded, each of which gives it value both in art and in science:—

1. The wave of the first order is to be regarded as a vehicle for the transmission of mechanical force (geological application).

2. The wave of the first order is an important element in the calculation and phenomena of resistance of fluids (form of ships, canals, &c.).

3. The wave of the first order is identical with the great oceanic wave of the tide (improvement of tidal rivers).

4. The water-wave of the first order presents some analogy to the sound wave of the atmosphere (phenomena of acoustics).

TABLE XVIII.

The Velocity of the Wave of the First Order, calculated for various depths of the fluid in a channel of uniform depth, extending a depth from 0·1 of an inch to 100 feet.

Column A contains the depths of the fluid in decimal parts of an inch.

Column B the corresponding velocities in feet per second.

Column C gives the depth in inches.

Column D the corresponding velocities in feet per second.

Column F gives the depths in feet.
Column G the corresponding velocities in feet per second.
Columns of Differences, E and H, will assist in extending the table.

A. Value of $h+c$ in inches.	B. Value of $\sqrt{g(c+h)}$ in ft. per sec.	C. Value of $h+c$ in inches.	D. Value of $\sqrt{g(c+h)}$ in ft. per sec.	E. First difference.	F. Value of $h+c$ in feet.	G. Value of $\sqrt{g(c+h)}$ in ft. per sec.	H. First difference.
0·0	0·0000	0·0	0·000		0·0	0·000	
·1	0·5179	1·0	1·637		1·0	5·674	
·2	0·7325	2·0	2·316		2·0	8·024	
·3	0·8971	3·0	2·836		3·0	9·827	
·4	1·0359	4·0	3·275		4·0	11·347	
·5	1·1581	5·0	3·662		5·0	12·687	
·6	1·2687	6·0	4·011		6·0	13·898	
·7	1·3703	7·0	4·333		7·0	15·011	
·8	1·4649	8·0	4·632		8·0	16·047	
·9	1·5538	9·0	4·913		9·0	17·021	
1·0	1·6378	10·0	5·179		10·0	17·942	
·1	1·7178	11·0	5·432	253	11·0	18·817	875
·2	1·7942	I. 12·0	5·673	241	12·0	19·654	837
·3	1·8674	13·0	5·905	231	13·0	20·457	803
·4	1·9379	14·0	6·128	222	14·0	21·229	772
·5	2·0060	15·0	6·343	215	15·0	21·974	745
·6	2·0717	16·0	6·551	207	16·0	22·695	721
·7	2·1355	17·0	6·753	201	17·0	23·393	698
·8	2·1974	18·0	6·948	195	18·0	24·071	678
·9	2·2576	19·0	7·139	190	19·0	24·731	660
2·0	2·3163	20·0	7·324	185	20·0	25·374	643
·1	2·3735	21·0	7·505	180	21·0	26·000	626
·2	2·4293	22·0	7·682	176	22·0	26·612	612
·3	2·4839	23·0	7·854	172	23·0	27·210	598
·4	2·5373	II. 24·0	8·023	168	24·0	27·796	586
·5	2·5896	25·0	8·189	165	25·0	28·368	572
·6	2·6409	26·0	8·351	162	26·0	28·930	562
·7	2·6913	27·0	8·510	159	27·0	29·481	551
·8	2·7405	28·0	8·666	156	28·0	30·023	542
·9	2·7891	29·0	8·820	153	29·0	30·554	531
3·0	2·8368	30·0	8·970	150	30·0	31·076	522
·1	2·8834	31·0	9·118	149	31·0	31·589	513
·2	2·9299	32·0	9·265	147	32·0	32·095	505
·3	2·9753	33·0	9·408	143	33·0	32·593	497
·4	3·0200	34·0	9·550	141	34·0	33·083	490
·5	3·0641	35·0	9·689	139	35·0	33·566	480
·6	3·1076	III. 36·0	9·827	137	36·0	34·042	476
·7	3·1505	37·0	9·962	135	37·0	34·512	470
·8	3·1928	38·0	10·096	133	38·0	34·976	464
·9	3·2337	39·0	10·228	131	39·0	35·434	458
4·0	3·2756	40·0	10·358	130	40·0	35·883	449
·1	3·3164	41·0	10·487	128	41·0	36·329	446
·2	3·3566	42·0	10·614	127	42·0	36·771	442
·3	3·3963	43·0	10·740	125	43·0	37·205	434
·4	3·4356	44·0	10·864	124	44·0	37·635	430
·5	3·4744	45·0	10·987	122	45·0	38·060	425
·6	3·5128	46·0	11·108	121	46·0	38·481	421
·7	3·5508	47·0	11·229	120	47·0	38·897	416
·8	3·5884	IV. 48·0	11·347	118	48·0	39·308	411

TABLE XVIII.—*continued.*

A. Value of $h+c$ in inches.	B. Value of $\sqrt{g(c+h)}$ in ft. per sec.	C. Value of $h+c$ in inches.	D. Value of $\sqrt{g(c+h)}$ in ft. per sec.	E. First difference.	F. Value of $h+c$ in feet.	G. Value of $\sqrt{g(c+h)}$ in ft. per sec.	H. First difference.
·9	3·6225	49·0	11·464	117	49·0	39·716	408
5·0	3·6623	50·0	11·581	116	50·0	40·119	403
·1	3·6988	51·0	11·696	115	51·0	40·518	399
·2	3·7348	52·0	11·810	114	52·0	40·913	395
·3	3·7704	53·0	11·923	113	53·0	41·304	391
·4	3·8056	54·0	12 035	112	54·0	41·693	389
·5	3·8405	55·0	12·146	111	55·0	42·079	386
·6	3·8758	56·0	12·256	110	56·0	42·458	379
·7	3·9101	57·0	12·365	109	57·0	42·834	376
·8	3·9441	58·0	12·473	108	58·0	43·209	375
·9	3·9778	59·0	12·580	107	59·0	43·580	371
6·0	4·0120	V. 60·0	12·686	106	60·0	43·948	368
·1	4·0451	61·0	12·791	105	61·0	44·315	367
·2	4·0779	62·0	12·895	104	62·0	44·678	363
·3	4·1105	63·0	12·998	103	63·0	45·037	359
·4	4·1434	64·0	13·101	103	64·0	45·392	355
·5	4·1755	65·0	13·203	102	65·0	45·745	353
·6	4·2074	66·0	13·305	102	66·0	46·095	350
·7	4·2390	67·0	13·406	101	67·0	46·442	347
·8	4·2710	68·0	13·506	100	68·0	46·786	344
·9	4·3021	69·0	13·605	99	69·0	47·127	341
7·0	4·3333	70·0	13·704	99	70·0	47·467	340
·1	4·3640	71·0	13·801	97	71·0	47·805	338
·2	4·3958	VI. 72·0	13·897	96	72·0	48·142	337
·3	4·4251	73·0	13·993	96	73·0	48·477	335
·4	4·4551	74·0	14·088	95	74·0	48·809	332
·5	4·4850	75·0	14·183	95	75·0	49·137	328
·6	4·5152	76·0	14·277	94	76·0	49·462	325
·7	4·5447	77·0	14·371	94	77·0	49·786	324
·8	4·5740	78·0	14·464	93	78·0	50·108	322
·9	4·6031	79·0	14·556	92	79·0	50·429	321
8·0	4·6325	80·0	14·648	92	80·0	50·748	319
·1	4·6612	81·0	14·739	91	81·0	51·061	317
·2	4·6898	82·0	14·830	91	82·0	51·376	315
·3	4·7182	83·0	14·921	91	83·0	51·689	313
·4	4·7470	VII. 84·0	15·011	90	84·0	52·000	311
·5	4·7761	85·0	15·100	89	85·0	52·309	309
·6	4·8040	86·0	15·189	89	86·0	52·616	307
·7	4·8318	87·0	15·277	88	87·0	52·921	305
·8	4·8586	88·0	15·364	87	88·0	53·224	303
·9	4·8860	89·0	15·451	87	89·0	53·526	302
9·0	4·9134	90·0	15·537	86	90·0	53·827	301
·1	4·9404	91·0	15·623	86	91·0	54·126	299
·2	4·9678	92·0	15·709	86	92·0	54·423	297
·3	4·9946	93·0	15·794	85	93·0	54·719	296
·4	5·0213	94·0	15·879	85	94·0	55·014	295
·5	5·0479	95·0	15·963	84	95·0	55·307	293
·6	5·0746	VIII. 96·0	16·047	84	96·0	55·597	290
·7	5·1011	97·0	16·130	83	97·0	55·886	289
·8	5·1275	98·0	16·212	82	98·0	56·172	286
·9	5·1538	99·0	16·293	81	99·0	56·455	283
10·0	5·1792	100·0	16·373	80	100·0	56·737	282

Section II.—WAVES OF THE SECOND ORDER.

Oscillating Waves.

Character	Gregarious.
Species	Stationary. Progressive.
Varieties	Free. Forced.
Instances	Stream ripple. Wind waves. Ocean swell.

The Standing Wave of Running Water.—Among oscillating waves of the second order, I know none more common or more curious than the standing wave of running water. I begin the account of my examination of waves of the second order, because it is that species which appears to me to be the most easy to be conceived, because it presents the closest analogy to the ordinary known phenomena of wave motion, and because, although most frequently exhibited to the eye of the common gazer, it has not, as far as I know, ever been made the subject of accurate observation.

If the surface of a running stream be examined as it runs with an equal velocity along a smooth and even channel, its surface will present no remarkable feature to the eye, although it is known by accurate observation that the surface of the water is higher above the level in the middle or deep part than at the sides of the channel. On the bottom of the channel let there be found a single large stone; this interruption, although considerably below the surface of the water, will give indication of its presence by a change of form visible on the surface of the water. An elevation of surface will be visible, not immediately above it, but in its vicinity. Simultaneous with the appearance of this protuberance, there will appear a series of others lower down the stream. These form a group of companion phenomena, are waves of the second order, oscillatory, and of the standing species, their place remaining fixed in the water, while the water particles themselves continue to flow down with the stream. For examples see Plate IX.

This species of wave is especially deserving of the notice both of the mathematician and of the natural philosopher, for this cause especially, that the apparent motions of the water are in this case identical with the actual paths of individual particles; each particle on the surface actually describes the path apparent on the surface; the outline of the surface of the water is the true path of a particle during its progress down the stream. It does not exhibit like other waves the form merely, a form very different from the true motion of the water particles, nor does it exhibit the motion of a motion, nor do the particles themselves remain behind while they transmit forward the wave. The particles are themselves translated along the fluid in the paths which form the apparent outline of the fluid.

In this respect, therefore, this wave appears to me important as presenting a case of transition from ordinary fluid motion to wave motion.

I found by observation on a mountain stream that waves 3½ feet long rise in water moving at the rate of 3½ feet per second.

Also, that waves 2 feet long rose in water moving at 2½ feet per second.

These numbers coincide with those given in Table XXI., from which the following approximate numbers are deduced. These numbers will enable an observer to judge of the velocity of a stream by inspection of the waves on the surface.

The length of wave	being	1 inch,	the velocity	of the stream	per second	is	½ foot.
"	"	*3 inches,	"	"	"	*1	"
"	"	1 foot,	"	"	"	1¾	feet.
"	"	1¼ feet,	"	"	"	2	"
"	"	2 "	"	"	"	2½	"
"	"	*3½ "	"	"	"	*3½	"
"	"	6 "	"	"	"	4½	"
"	"	7 "	"	"	"	5	"
"	"	10 "	"	"	"	6	"
"	"	*30 "	"	"	"	*10	"

This table is given for convenience of reference to observers, and it is useful and easy to recollect the velocities corresponding to 3 inches, 3½ feet, and 30 feet. By these means it will be easy for observers to verify or correct these numbers.

These waves are very peculiar in this respect, that they exhibit little or no tendency to lateral diffusion; the breadth of a wave does not apparently exceed the length of a wave, and is often much smaller. When a stream enters a large pool, its path across the pool is marked by these waves very distinctly, and the diminishing length of the waves accompanies the diminishing velocity of the stream, and at the same time indicates the extreme slowness with which diffusion takes place.

The motion of the particles of water, as observed by a body floating on the surface, is this, the motion is retarded at the top of each wave and accelerated in the bottom, thus oscillating about the mean motion of the stream. The motion, as far as it can be observed by bodies floating near the surface, is a simple combination of a circular with a rectilineal motion. The disturbing body, the stone at the bottom, gives to the particles which pass over it the motion of eddy as indicated, Plate IX. fig. 2, and this being continued downwards, and combined with the rectilineal motion of the particles, presents the cycloidal form of the wave.

If we conceive a uniform revolving motion in a vertical plane communicated to a particle of water, the centre of the circle of revolution being at the same time carried uniformly along the horizontal line, Plate X., then the path of the particle having these two motions is marked out by the cycloidal line 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12 joining these points, and if every successive particle of the fluid have the same motions communicated to it, the simultaneous places of successive particles will give the line 1, 2, 3, 4, 5, 6, 7, 8, &c., as the form of the surface of the fluid. It is to be observed that at A and C the direction of the motion of revolution is opposite to the motion of transference, and ∴ the absolute velocity of the particle is diminished by the oscillating motion, while at B and D it is increased by an equal amount, and in the intermediate positions 3 and 9 it is neither increased nor diminished. It is also to be observed, that when the motion of the water in the direction of transference is slowest (*i.e.*, when the motion of oscillation is opposite to the motion of transference), the transverse section of moving fluid is greatest, and when the motion of transference and of oscillation coincide, and the motion is quickest in the direction of transference, the transverse section of the fluid is greatest. Thus we see how during a change of form the dynamical equilibrium of the fluid may be unchanged.

The fluid may thus be conceived as moving with varying velocity along a channel of variable section, its upper surface being conformable to the outline of the wave. Hence we might infer that a rigid channel of varying area, of the form of this standing wave, would not interfere with the free motion of the fluid.

And hence it may follow, that when the area of a pipe conveying fluid is to undergo a change, the best form of pipe or channel is indicated by the form of this wave. Thus the velocity has undergone a change between 0 and 4 which the form of a close pipe might render permanent.

In the examples already given, a solid impediment has generated the waves on the surface of the fluid. At the confluence of streams I have observed the same waves generated by the oblique action of one current on another meeting it in a different direction.

The height and hollow of the fluid and the change of velocity are to be regarded as reciprocally the cause and effect each of the other. The obstacle first retards the velocity of the fluid, so as to accumulate it above the obstacle, the water rises to a height due to this diminished velocity, and as all the particles of the stream pass through this area of the stream with a diminished velocity, the area of transverse section must be increased at this point; thus the elevation of surface, enlargement of section, diminution of velocity above the obstacle, are its necessary consequences of that obstacle. Again, below the obstacle the accumulation above generates an additional velocity due to that height in addition to the mean motion of the stream; the same volume of water which passed through the large area, with its increased section and diminished velocity, being now a higher velocity, is transferred through the smaller area which allows its transmission. Thus the constant volume passing down the stream varies its velocity with the conservation of its forces by means of a varying area of transference; and thus we are enabled to conceive how the observed form of the surface becomes at once possible and necessary to the transmission of the fluid under the action of the disturbing force.

I am not aware that this species of standing wave in moving water has ever before been made the subject of philosophical examination. But I conceive that its study is highly important, especially in a theoretical view, as the means of conveying sound elementary conceptions of wave motion, as exhibiting the transition from the phenomena of water currents to those of water waves, as the intermediate link between motions of the first degree and motions of the second degree, and as affording a basis from which we may commence, with some prospect of success, the application of the known principles and laws of motion to the investigation of the difficult theory of waves.

Moving Waves of the Second Order—Sea Waves.—It is not difficult to pass from the conception of standing waves in running water to the conception of running waves in standing water. Let us first conceive the waves in Plate IX. to be formed in water running in the direction there indicated from right to left, with a given mean motion, and a given motion of uniform circular oscillation: and next let us conceive the whole water channel and waves to be transferred uniformly in the opposite direction with a velocity equal to the mean velocity of transference; then the absolute motion of transference of the water will become nothing: the waves formerly standing are now moved in the opposite direction with a velocity equal to the former mean velocity of the running stream, and the motion of oscillation remains. Thus, the running water becoming still, the waves become moving waves, and if we reverse the hypothesis once more, and conceive the waves which move with a given velocity to exist in water which has a motion of transference with equal velocity in the opposite direction, it is manifest that these waves running up the stream as fast as the waters run down, the wave-crests remain fixed in place. Thus then the same oscillating phenomenon which in standing water gives moving waves, will give in moving water standing waves; taking for granted always that the motions of oscillation are such as to be possible, consistent with the nature of the fluid, and independent of the common mean motion of the fluid; a condition equally essential to the possibility of the wave motion and of our conceptions of it.

I have been able accurately to observe the phenomena of wave motion in still water, the waves being of the second order and gregarious, under the following circumstances:—

1. I have drawn a body through the water with a uniform motion, and have observed the group of waves which follow in its wake.

2. I have propagated the negative wave of the first order, and observed the group of waves which follow in its wake.

I have not observed in the results of these two methods any distinction of form, velocity, or other character.

The form under which these waves appear has already been exhibited in Plate VI. figs. 9 and 10, and equally in Plate IX. figs. 1, 2, 3, and in Plate X. fig. 1.

I have made a series of observations by dragging a body through the water, the results of which are given in the following table. I first made preparatory observations to find whether the form of body or depth of channel made any change on the phenomenon. I found that larger bodies and higher velocities made higher waves, but that the length and velocity of the wave were unchanged by either the form of body, or the depth of the channel, or the height of the wave. I observed that when the waves became high and broke, the elevation above the mean level was 6 inches, and the depression below it 2 inches, making a height total of 8 inches; this was at a velocity of 6·25 feet per second. Immediately behind the body dragged through the water, the mean level appears to be considerably lowered.

I examined the motion of oscillation of these waves by means of small floating spherules. Waves of the second order having a total height of half an inch, in water 4 inches deep made by a negative wave, were accompanied by motion in a circle of half an inch diameter at the surface, and the particles below described also circles which rapidly decreased in diameter and at 3 inches deep ceased to be sensible; the waves were about one foot long.

Table XIX.

Observations on the Length and Velocity of Waves of the Second Order.

Column A the order and number of the experiments.

Column B the number of seconds in which the waves were transmitted along 100 feet.

Column C the aggregate length in feet of the number of waves in Column D.

Column D the number of waves extending to the length in Column C.

Column E the length in feet of one wave from crest to crest.

Column F the velocity in feet per second given by experiment.

A.	B.	C.	D.	E.	F.	A.
I.	33·2	26·5	10	2·65	3·01	I.
II.	33·2	26·5	10	2·65	3·01	II.
III.	31·6	25·	8½	2·94	3·16	III.
IV.	31·8	25·	8½	2·94	3·14	IV.
V.	31·8	25·	8½	2·94	3·14	V.
VI.	30·4	25·	8	3·125	3·29	VI.
VII.	29·6	25·	7⅔	3·26	3·37	VII.
VIII.	29·6	25·	7⅔	3·26	3·37	VIII.
IX.	28·0	25·	7	3·57	3·57	IX.
X.	28·4	25·	7	3·57	3·51	X.
XI.	28·0	25·	7	3·57	3·57	XI.
XII.	28·0	25·	7	3·57	3·57	XII.
XIII.	28·0	25·	7	3·57	3·57	XIII.
XIV.	28·0	25·	7	3·57	3·57	XIV.
†XV.-XVII.	26·8	25·	6½	3·84	3·72	XV.-XVII.
†XVIII.-XXII.	26·0	25·	6	4·18	3·84	XVIII.-XXII.
†XXIII.-XXVI.	24·0	25·	5	5·00	4·16	XXIII.-XXVI.
†XXVII.-XXXIV.	21·6	25·	4	6·25	4·62	XXVII.-XXXIV.

These results are the means of many experiments, differing from each other not more than the examples preceding them, which have been given in detail as a fair specimen.

As these waves appear in groups, their velocity and lengths are easily observed and measured. I have reckoned as many as a dozen such waves in a group all about the same magnitude, so that the aggregate length of the first six was sensibly equal to the length of the second group of six. The method of observation was this: a given distance was marked off along one side of the channel; an observer marked the instant at which the first of a group of secondary waves arrived at a given point, while another observer at the further end of the given distance counted the number of waves as they passed, and marked the point at which the last had arrived when the signal was given that the first wave had reached the other station; thus it was observed that in a group of waves moving over 100 feet in 28 seconds, there were seven comprehended in a distance of 25 feet, whence

$\frac{28}{100} = 3{\cdot}57$ feet per second for the velocity of the wave, and

$\frac{25}{7} = 3{\cdot}57$ feet as the length of the wave.

Also, since the wave passes along 3·57 feet its own length in one second, its length divided by the velocity gives 1 second as the period of one complete oscillation.

The *velocity* of the wave of the second order, the *length* from the crest of one wave to the crest of the next, or from hollow to hollow, and the time of passing from one crest to another, called the *period* of the wave; these are the principal elements for observation.

These elements are calculated for the convenience of observers in the Table XXI. It will also be observed that the circles which represent the oscillatory motion of the water particles (Plate X.), showing the Wave Motion of the Second Order, diminish very rapidly with the increasing depth of the particles below the surface of the water at the lowest part of the wave. By my observations I found that in high waves at a depth $= \frac{1}{3}$rd of a wave length, the range of oscillation of the particles is only about $\frac{1}{30}$th of that of particles on the surface.*

* I have here to express the favourable opinion which I have formed of a wave theory given to the world by M. Franz Gerstner, so early as 1804,† and reprinted

† Theorie der Wellen. Prague, 1804.

One observation which I have made is curious. It is, that in the case of oscillating waves of the second order, I have found that the motion of propagation of the whole group is different from the apparent motion of wave transmission along the surface; that in the group whose velocity of oscillation is as observed 3·57 feet per second, each wave having a seeming velocity of 3·57, the whole group moves forward in the direction of transmission with a much slower velocity. The consequence of this is a difficulty

in the work of the MM. Weber, to whom I am indebted for my acquaintance with this theory. Gerstner's theory is characterised by simplicity of hypothesis, precision of application, its conformity with the pheuomena, and the elegance of its results. It is not without faults, yet I cannot agree with the Messrs. Weber, nor with MM. Professors Mollweide and Mobius, in the precise opinion at which they arrive, although I confess I could wish that he had assumed as an hypothesis the doctrine which in (14.) he deduces as a conclusion from hypotheses less firmly established than this conclusion, unless indeed we should esteem it an argument in favour of his hypothesis, that it conducts him directly to a conclusion of well-known truth. Neither do I find that his hypotheses are so much at variance with the actual conditions of the waves I have observed, as they appear to be in MM. Weber's view of their own experiments. The calculations of M. Gerstner are applied primarily to a kind of standing oscillation. But it does not appear to me that his calculations ought to be applied in any way to the standing oscillations which MM. Weber reckons to be their closest representation. In M. Gerstner's first part of the work the wave form is standing, wave oscillation is circular, the fluid is in motion, and the particle paths are identical with the lines which indicate the form of the wave. I conceive, therefore, that the wave which he has examined, and the conditions of its genesis, find a perfect representative in my standing waves of the second order, in running water, which I have represented in Plates IX. and X. From this hypothesis it is not difficult to arrive at the moving wave of standing water, for if we conceive the whole channel moved horizontally along in an opposite direction with a velocity equal to the horizontal velocity of transference, the particles will then be relatively at rest, the cycloidal waves become moving forms, the particle paths stationary circles, and the motion of transmission of the wave equal and opposite to the former mean horizontal motion of transference of the particles. In short, they become moving waves of the third order, the common waves of the sea.

From M. Gerstner's investigations we obtain the following results, for oscillating waves which correspond to our second order:—

1. Waves of the same amplitude are described in equal times independently of their height. (This corresponds with the results of our experiments.)

2. Waves are transmitted with velocities which vary as the square roots of their amplitudes.

3. The waves on the surface are of the cycloidal form, always elongated, never compressed; the common cycloid being the limit between the possible and impossible, the continuous and the broken wave.

4. The particle paths in the standing waves of running water are cycloids, which on the surface are identical with the wave form, and below the surface have the same character with the wave lines of the surface, the height of the waves only diminishing with the increase of depth.

5. The particle paths of moving waves in standing water are circles corresponding to the circles of height of the cycloidal paths; the diameters of these circles of vertical oscillation diminish in depth as follows. Let $0, u, 2u, 3u$, &c. be depths increasing in arithmetical progression, then $b, b\,\epsilon^{-\frac{u}{a}}, b\,\epsilon^{-\frac{2u}{a}}, b\,\epsilon^{-\frac{3u}{a}}$, which decrease in geometrical proportion, are the ratio of the diminishing diameters of vertical oscillation. Thus, if $0, \frac{1}{2}a, \frac{2}{2}a, \frac{3}{2}a$, &c., be depths, a, $0·6065\,a$, $0·3679\,a$, $0·2231\,a$, $0·1353\,a$, are the ranges.

6. The forms of these paths and the circles of oscillation are shown in Plate X. fig. 1, which has been drawn with geometrical accuracy from the data of M.

in observing these waves (especially such as are raised by the wind at sea), namely, that as the eye follows the crest of the wave, this crest appears to run out of sight, and is lost in the small waves in which the group terminates. The termination of these groups in a series of waves becoming gradually smaller and smaller, yet all continuous with the large wave, is curious and leads to a curious conclusion. It is plain that if these large waves are moving with the same velocity as the small ones, this result would be inconsistent with the other experiments. But if we conceive each to be transmitted with the velocity due to its breadth, we shall have the velocity of oscillation varying from point to point in the same group of waves, but it will be impossible always to measure this velocity directly as it may be continually changing. There is to be observed, therefore, this distinction in a group of waves of the second order, between the velocity of individual wave transmission and the velocity of aggregate wave propagation.

I have not found it possible to measure this velocity of aggregate propagation of a group of waves, from want of a point to observe. If I fix my eye upon a single wave, I follow it along the group, and it gradually diminishes and then disappears; I take another and follow it, and it also disappears. My eye, in following a wave crest, follows the visible velocity of transmission merely. After one or two such observations, I find that the whole group of oscillations has been transferred along in the direction of transmission with a velocity comparatively slower; but I have not been able to measure this velocity of propagation of the wave motion from one place to another.

We have already seen that the velocity assigned by Mr. Kelland and Mr. Airy falls much short of that of the wave of the first order, to which they have thought their results were to be applied. Their results are much nearer to that of the secondary wave, so that it may be questioned whether they should not have

Gerstner's theory, and it is at the same time the most accurate representative I am able to give of my observations on the wave of the second order.

7. The period of wave oscillation is $t = \pi \sqrt{\frac{2a}{g}}$.

8. The velocity of wave propagation is $v = \sqrt{2ag}$, a being the radius of the wave cycloid generating circle.

9. It follows that the length of a pendulum isochronous with the wave is less than the wave length in the ratio of the diameter of a circle to its semi-circumference. Newton made these equal. These last three results are inconsistent with my observations on transmission.

applied their results to that rather than the other. Thus by comparing Table XXI. with Table XVIII., it will be found that while the velocity of a wave of the first order, about 6 feet long, is from 5·5 to 8 feet per second, according to the height, that of a wave of the second order is only 4·62 feet, which is much nearer to their results. There remains however this difficulty, that high and low waves of the second order of equal length have equal velocities.

On Observations of the Waves of the Sea.—The chief difficulty in obtaining accurate measures of sea waves consists in this fact, that the surface is seldom covered with a uniform series of equidistant equal waves, but with several simultaneous groups of different magnitude or in different directions. If there exist more groups than one, the resulting apparent motion of the surface will be extremely different from the motion of either, and may be apparently in an opposite direction from that of the actual motion of the individual series themselves.

Besides the coexistence of different series of waves, we have the difficulty arising from the fact already mentioned, that a difference exists between the velocity of transmission and the velocity of propagation. From this it results, that after the eye has followed the apparent ridge of a wave, moving with a given velocity of transmission, it will outrun the velocity of propagation, and the wave will appear to cease. This I have continually observed at sea. The eye follows a large wave and suddenly it ceases to pass on, but on looking back we find it making once more an appearance on the same ground along which we formerly traced its ridge; this arises from the cause just mentioned.

But there are still many occasions on which tolerable observations may be made, and the best will be such as are least complicated by separate systems. The best observations of this kind I have been able to obtain were made for the Committee of the British Association, by the Queen's Harbour-master at Plymouth, William Walker, Esq., who has paid much attention to this subject. He observed the waves as they traversed a space of about half a mile, between two buoys, noting the time of passing, and also the number of waves in the distance between the buoys, whose distance was accurately known. He remarks that in counting the number of waves, great difficulty was found in following a single wave along this space. In fact, as we have already shown, a wave will be often found to fall behind its expected place.

The resulting velocities got from Mr. Walker's experiments are very various. But on taking out of the others all those which are mentioned by Mr. Walker as having causes of uncertainty, I found those which remained very close to those given in Table XXI.

The following is the Table of observations on sea waves.

Distance traversed about half a mile; depth 40 to 50 feet.

TABLE XX.

Observations on the Length and Velocity of Waves of the Second Order.—In the Sea.

	Wave length.	Velocity per sec.	Velocity per hour.	Height of wave in feet above mean level.	Remarks at the time of Observation.
	Feet.	Feet.	Miles.		
I.	110·5	20·2	11·9	2¾	A fresh breeze blowing.
II.	175·0	34·3	20·3	2½	Waves not easily traced.
III.	302·	37·0	21·9	4	High seas overtake smaller ones.
IV.	345·	37·0	21·9	4½	These waves came down channel.
V.	·306·	37·0	21·9	4½	Long low swell.
VI.	408·	41·2	24·2	4¼	Small waves merged in large ones.
VII.	442·	41·8	24·7	27	Height of wave correctly measured, they break in 5 and 6 fathoms water.
VIII.	450·	44·7	26·5	?	Strong S.W. wind.
IX.	460·	46·0	27·2	?	Waves running high and breaking.
X.	345·	46·0	27·2	5	Long low swell.
XI.	394·	38·3	22·7	5	Waves generated by wind of yesterday.
XII.	345·	41·5	24·5	4	Waves crowd near the beach.
XIII.	306·	36·8	21·6	irregular.	Shifting wind.
XIV.	460·	42·5	25·2	regular.	Easterly winds.

Of these there are five which coincide with my observations and with my tables, Nos. XIX. and XXI.; and it is curious that these five are those which are made in the most unexceptionable circumstances. No. II. has the remark that the waves are not easily traced. No. III. has a mixture of waves, which always causes great confusion and difficulty of observation. No. V. and No. X. are long and low, and therefore not easily traced, and so on; but Nos. I., IV., VII., XI., XIV., are unexceptionable, and are compared with my formula in the following Table:—

	Length of wave observed.	Velocity of wave observed.	Velocity of wave calculated.
	Feet.	Feet per sec.	
I.	110·5	20·2	19·5
IV.	345·	37·0	35·
VII.	442·	41·8	40·
XII.	394·	38·3	37·
XIV.	460·	42·	40·*

We may therefore continue to use Table XXI. for the velocity of sea waves, unless we obtain further and decisive experiments to the contrary. It does not appear that sea waves present any characteristic to distinguish them from other oscillating waves of the second order which I have experimentally examined.

It also follows that these waves coincide with my observations, that the depth of water is the limit of the height of waves; see No. VII., where waves 27 feet high, break in water of 5 to 6 fathoms.

How it happens that individual large waves should ever arise in the surface of a large sea, uniformly exposed to the action of the wind, is not very obvious. Thus much is plain—that if a wave, greater than those around it, be generated by a local inequality of the wind, or by one of the moving whirlpools which we know to be so common, *that wave* will be increased continually by the presence of other waves coexisting with it, for when these other waves are crossing the top of this larger wave, they are suddenly exposed to increased force by the obstruction they present to the wind, and being cusped in form by the coincidence of the crests, they are in a position of delicate equilibrium easily deranged; and the derangement producing a breaking of the wave, the disintegrated fragments of the smaller wave detached from it, leave it smaller, and increase by an equal quantity the magnitude of the larger.

This exaggeration of an individual wave or group is increased by the phenomenon already noticed, that the velocity of wave *transmission* may be very different from the velocity of wave *propagation*. A large wave of the sea remaining in a state of much slower motion than the motion of wave transmission, being traversed by another series of different velocity, exposes them successively on its summit to the increased action of the wind to disintegration, thus making them tributary to its own further accumulation; such phenomena I have often noticed at sea; the wave appears to over-run itself; and the wave behind *seems* to take its place and acquire the magnitude and form it has appeared to lose; but it is the same wave which remains behind it, and its motion is merely a deception, or rather it is as explained in a preceding paragraph.

The final destruction of the waves of the sea, as they expend their strength and conclude their existence on the rocks and sands of the shore, is a subject of interesting study and observation. The sea-shore after a storm is a scene of great grandeur; it presents an instance of the expenditure of gigantic forces, which impress the mind with the presence of elemental power as sublime as the water-fall or the thunder. It is peculiarly instructive to watch these waves as they near the shore; long before they reach the shore they may be said to feel the bottom as the water becomes gradually more shallow, for they become sensibly increased in height; this increase goes on with the diminution of depth and a diminution of length likewise as the wave becomes sensible; finally, the wave passes through the successive phases of cycloidal form, as in Plate X., and becoming higher and more pointed, reaching the limit of the cycloid, assumes a form of unstable equilibrium, totters, becomes crested with foam, breaks with great violence, and continuing to break, is gradually lessened in bulk until it ends in a fringed margin on the sea-shore.

But there are a variety of questions to be determined concerning this shore wave or breaking surf. Why and how does it break? What happens after it begins to break? What are the relative levels of the waves and of the water? What is the mean level of the sea, and what sort of waves are breakers?

It is not at first obvious what form the mean level of the sea will assume on a sloping beach sea-ward on which heavy breakers are rolling. It is plainly not level; the action of the wind is known to heap the water up on it. The impetus of the waves also must raise it to some height due to their velocity and force. Hence the mean surface of the sea will form a slope upwards towards the sea-shore; and this slope will form a continual and uniform current of water outwards towards the sea, except when it is directly opposed by the action of the wave in the opposite direction.

There is a phenomenon of some importance in breaking waves, to which I have directed attention; it is this, that the wave of the second order disappears, and that a wave of the first order takes its place. It is to be observed as follows:—In waves breaking on a shore, I have observed a phenomenon which is curious and not without importance. The wave of the second order may disappear, and a wave of the first order take its place. The conditions in which I have noticed this phenomenon are as follows. One of the common sea waves, being of the second order, approaches the shore, consisting as usual of a negative or hollow part, and of a positive part elevated above the level; and as formerly noticed, this positive portion gradually increases in height, and at length the wave breaks, and the positive part of the wave falls forward into the negative part, filling up the hollow. Now we readily enough conceive that if the positive and the negative part of a wave were precisely equal in height, volume, and velocity, they would, by uniting, exactly neutralise each other's motion, and the volume of the one filling the hollow of the other, give rise to smooth water; but in approaching the shore the positive part increases in height, and the result of this is, to leave the positive portion of the wave much in excess above the negative. After a wave has first been made to break on the shore, it does not cease to travel, but if the slope be gentle, the beach shallow and very extended (as it sometimes is for a mile inwards from the breaking-point, if the waves be large), the whole inner portion of the beach is covered with positive waves of the first order, from among which all waves of the second order have disappeared. This accounts for the phenomenon of breakers transporting shingle and wreck, and other substances shorewards after a certain point; at a great distance from shore or where the shores are deep and abrupt, the wave is of the second order, and a body floating near the surface is alternately carried forward and backward by the waves, neither is the water affected to a great depth; whereas nearer the shore, the whole action of the wave is inwards, and the force extends to the bottom of the water and stirs the shingle shorewards; hence the abruptness also of the shingle and sand near the margin of the shore where the breakers generally run.

I have observed this most strikingly exemplified in Dublin Bay after a storm: there is a locality peculiarly favourable to the study of breaking waves above Kingston, where over an extent of several miles there is a broad, flat, sandy beach, varying in level very slightly and slowly. Waves coming in from the deep sea are first broken when they approach the shallow beach in the usual way; they give off residuary waves, which are positive; these are wide asunder from each other, are wholly positive, and the space between them, several times greater than the amplitude of the wave, are perfectly flat, and in this condition they extend over wide areas and travel to great distances. These residuary positive waves evidently prove the existence, and represent the amount of the excess of the positive above the negative forces in the wind wave of the second order. See Plate III. fig. 7.

TABLE XXI.

Length, Period, and Velocity of Transmission of Waves of the Second Order.

A the length of the waves (observed) in feet.

B the period of the waves in seconds.

C the velocity of the waves in feet per second (by observation).

D the velocity of the waves in feet per second, calculated by formula.

A.	B.	C.	D.	A.	A.	B.	C.	D.	A.
0·01	·053		·1889	0·01	8	1·496		5·344	8
0·05	·118		·4224	0·05	9	1·587		5·667	9
0·1	·167		·5975	0·1	10	1·670		5·975	10
0·25		1·00			20	2·366		8·45	20
0·3	·290		1·034	0·3	30	2·90		10·34	30
0·5	·374		1·336	0·5	40	3·34		11·95	40
0·7	·443		1·580	0·7	50	3·74		13·36	50
1·0	·529		1·889	1·0	100	5·29		18·89	100
1·2	·579		2·070	1·2	110		20·	19·5	
1·5	·648		2·314	1·5	200	7·48		26·72	200
1·7	·690		2·463	1·7	300	9·16		32·73	300
2·0	·748		2·672	2·0	345		37·	35·	
2·2	·781		2·802	2·2	394		38·	37·	
2·4	·820		2·927	2·4	400	10·58		37·78	400
2·65	·862	3·01	3·075	2·65	442		42·	40·	
2·94	·907	3·15	3·240	2·94	460		42·	40·	
3·00	·916		3·273	3·00	500	11·83		42·25	500
3·12	·934	3·29	3·338	3·12	1,000	16·70		59·75	1,000
3·26	·955	3·37	3·411	3·26	2,000	23·66		84·5	2,000
3·57	1·000	3·57	3·57	3·57	3,000	29·0		103·4	3,000
3·84	1·038	3·72	3·702	3·84	4,000	33·4		119·5	4,000
4·00	1·058		3·778	4·00	5,000	37·4		133·6	5,000
4·18	1·095	3·84	3·909	4·18	10,000	52·9		188·9	10,000
4·50	1·122		4·008	4·50	20,000	74·8		267·2	20,000
4·70	1·147		4·096	4·70	30,000	91·6		327·3	30,000
5·00	1·183	4·16	4·225	5·00	40,000	105·8		377·8	40,000
6·00	1·296		4·628	6·00	50,000	118·3		422·5	50,000
6·25	1·323	4·62	4·724	6·25	100,000	167·0		597·5	100,000
6·5	1·349		4·817	6·5	500,000	374·0		1336·0	500,000
7	1·400		4·999	7	1,000,000	529·0		1889·0	1,000,000

Section III.—WAVES OF THE THIRD ORDER.

Capillary Waves.

Character	Gregarious.
Varieties	Forced. Free.
Instances	Dentate waves. Zephyral waves.

Capillary Waves.—If the point of a slender rod or wire, being wet, be inserted in a reservoir of water perfectly still, to a minute depth, say one-tenth part of an inch below the surface of repose, it is known that the surface of the water will visibly rise in the vicinity of this wire, being highest in the immediate vicinity of the wire, and gradually diminishing until it cease to be sensible. I have examined this elevation by reflected rays from the surface, and I find that this elevated mass does not sensibly rise from the surface at more than an inch distance from the centre of the rod, the rod itself being one-sixteenth of an inch in diameter.

This statical phenomenon belongs to a well-known class of phenomena, which have been experimentally examined by many philosophers, and successfully explained by Dr. Thomas Young and Laplace, and recently investigated very fully and completely by M. Poisson, in his profound work entitled, "Nouvelle Théorie de l'Action Capillaire," Paris, 1831. An admirable report on the present state of our knowledge of the phenomena of capillary attraction will be found in the Transactions of the British Association, vol. ii. All that it is necessary for my present purpose to advert to on this subject is, that the phenomena of elevation of fluids by capillary attraction, are chiefly due to the condition of tension of the superficial particles of the water under the influence of a force acting on these superficial particles at insensible distances only, or by physical contact or adhesion. These superficial particles form a chain, or catenary, or lintearian curve, one end supported by the immediate adhesion of one extremity to the solid body at a given height above the water, the other end lying on the surface of the water, the underlying particles being suspended immediately by their mutual adhesion to this superficial film. M. Poisson especially has shown that "capillary phenomena are due to molecular action, modified by a particular state of compression of the fluid at its superficies." I have been thus particular for the purpose not only of explaining my meaning in a future article, but also to justify a term which I am desirous of introducing here as an expression not only convenient, but also philosophically sound. I have called the phenomena noticed in this section *Capillary Waves,* because they appear to me to present themselves exclusively in the thin superficial film which forms the bounding surface of the free liquid, and which is already recognised in the known hydrostatical phenomena of capillary attraction, and which, if I may be allowed, I will call the *capillary film.*

By capillary waves I therefore designate a class of hydrodynamical phenomena, which exhibit themselves when particles of water are put in motion under the action of such forces as when at rest produce the usual hydrostatical capillary phenomena. Let the slender rod already alluded to, as supporting a capillary column, bounded by a concave surface of revolution, be moved horizontally along the surface of the fluid with a velocity of one foot per second, and we shall have exhibited to us all the beautiful phenomena represented in Plate XI. In order to produce these phenomena, it is only necessary that the slender rod touch the surface without descending to any sensible depth; and the depth to which it descends in no sensible manner affects the phenomenon.—I have called these phenomena capillary waves.

Free Capillary Waves.—If the point of a rod sustaining a capillary column be suddenly raised, so as to allow the capillary film to remain without support, it descends and propagates through the capillary film an undulation which diffuses itself in every direction circular-wise, in a small group of about half a dozen visible waves which soon become insensible. Or if a very slender silk fibre, stretched horizontally along the surface of the water, be first wetted, and made to sustain a long strip of the capillary film, and then suddenly withdrawn, leaving a ridge of unsupported fluid, waves parallel to this are generated, which remain longer visible, are short and narrow at first, and becoming longer and flatter, at first about a quarter of an inch in amplitude from ridge to ridge, and about half a dozen in number, they become an inch in amplitude about the time when they are last visible; their *longevity* does not exceed *twelve or fifteen seconds,* and their *visible range eight or ten feet.*

These latter are what I designate the *free* capillary waves; the former class, shown in Plate XI., existing under the continued influence of the disturbing force, may be called the *forced* species of this order of wave. As forced waves, and while under the influence of the exciting body, they may apparently attain great velocity; but if the disturbing body be suddenly removed, they immediately expand backwards from the place where they were crowded by the solid point, and becoming all of nearly equal breadth, move forward together as free waves for twelve or fifteen seconds, at a rate of 8½ inches per second.

Forced Capillary Waves.—I have already stated that if a slender rod or wire, one-sixteenth of an inch in diameter, be inserted, after having been wetted, into water in repose, there will be raised all round this rod a column of fluid by the action of the capillary forces, as indicated at fig. 2, Plate X. I have stated that this surface may be observed by reflection to extend on every side about an inch, forming a circular elevation, bounded by a surface of revolution round the axis of the rod as a centre; the line which divides the elevated from the level surface being a circle of two inches in diameter. When this rod is moved horizontally along the surface of the fluid, the form of the elevated mass changes; before the disturbing point the extent of elevation diminishes, and the outline of the capillary volume of fluid sustained by the cylinder ceases to be a figure of revolution, becoming distorted as at fig. 3. At a velocity of about eight inches per second, the capillary volume has taken the bifurcate form, fig. 6, and a small wave, *b b*, about an inch broad, is visible before the disturbing point, and a ridge, *a a*, begins to manifest itself, diverging from the disturbing body; at about ten inches per second there become visible distinctly three waves, the disturbing body being in the middle of the first *a*, and the sum of the length of waves *b* and *c*, being about an inch. At higher velocities than this, the waves increase rapidly in number, diminish in amplitude, and extend out in length, spreading into the form indicated in Plate XI., which is formed at a velocity of 60 feet per minute, or of 12 inches per second.

As the velocity increases, the following changes are to be observed:—1. The waves diminish in amplitude from ridge to ridge; that is to say, denominating the wave in which is the disturbing body ridge *a*, and the others in succession before the point *b c d*, &c., the first space of an inch forward, in the direction of motion contains at a velocity of 12 inches per second, or

60 feet per minute, besides *a*, 3 ridges *b c d;* at 65 feet per second 4 ridges *b c d e;* at 72 feet per second there are in the first inch formed five ridges *b c d e f*, and so on. This crowding of the ridges with the velocity is given in the following Table :—

TABLE XXII.

Observations on the Velocity, Distance, and Divergence of Waves of the Third Order.

Column A contains the time in which the disturbing body, a wire of one-sixteenth of an inch in diameter, was drawn with a uniform motion along distances of 12 feet each; each experiment being frequently repeated.

B and C are the corresponding velocities of the disturbing body.

D, E, F are the number of complete waves, reckoning from hollow to hollow, contained in each successive inch from the centre of the disturbing wire, formed in the direction of the motion of the wire.

G. The numbers in this column are measures of the divergence of the first wave from the path of the exciting wire, measured at 25 inches behind that wire, and of course these numbers are tangents to the radius 25 for the angle of divergence.

H contains the angles deduced from the numbers in G.

Observations on the Capillary Waves.

See Plate XI.

	A.	B.	C.	D.	E.	F.	G.	H.
				No. of waves observed before the disturbing body.				
	Time of describing 12 feet.	Velocity in feet per sec.	Velocity in feet per min.	in 1st inch.	in 2nd inch.	in 3rd inch.	Tang. of angle to radius 25.	Angle of crest of first wave *aa*, with direction of disturbing bodies.
	s.							°
I.	12	1	60	3	4	5 ?	25	45
II.	11	$1\frac{1}{11}$	65	4	5	6 ?	21	40
III.	10	$1\frac{2}{10}$	72	5	6	7 ?	17	34
IV.		$1\frac{3}{9}$	80	6	7		14	29
V.	8	$1\frac{4}{8}$	90	7	8		11	24
VI.	7	$1\frac{5}{7}$	103	8			9	20
VII.	6	2	120	9			8	18
VIII.	5	$2\frac{1}{5}$	132				7	
IX.	4	3	180				6	

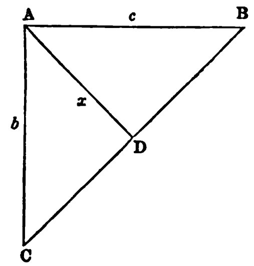
The crowding of the ridges is not the only phenomenon that accompanies the increase of velocity of the moving point; the first wave, that whose ridge is in the focus, scarcely differs from a straight line, and the angle which it makes with the path of the disturbing point, diminishes with the increase of velocity; the divergence of the first wave from the path of the exciting body is given in another column by an observation of the distance of the wave from that path at a given distance behind the body. These numbers show that the velocity of the wave, taken at right angles to the ridge, is nearly that of the free wave. This angle therefore becomes an index of the relation of the velocity of disturbance to the velocity of wave propagation.

The form of the wave ridges appears to be nearly that of a group of confocal hyperbolas, the exciting body being in the focus.

I have found the numbers given in columns C, D and E, to be determined by the velocity of the disturbing body, and quite independent of its size and form. But while I have found the number of ridges in an inch at a given velocity to be thus invariable, I have not found the number of inches over which these vibrations range to be equally invariable. At a velocity of 100 feet per minute, they may sometimes be observed advancing only over the first two inches before the point; then suddenly the vibrations will spread out, not increasing in magnitude but in number to thirty or forty, extending along many inches in advance of the disturbing point, and covering ten or twelve square feet with an extension of the representation in Plate XI. Then suddenly without apparent cause, they will subside and become visible only as a thin narrow belt, comprising the two or three waves nearest the disturbing body, and as suddenly will again spread out over the surface of the water. The play of this beautiful symmetrical system of confocal hyperbolas is a phenomenon not inferior in beauty to some of the exquisite figures exhibited by polarising crystals. I have found that the purity of the water had much to do with the extent and range of this phenomenon; that any small particles loading at a few points the capillary film was sufficient to derange the propagation of these waves, and prevent their distribution over a wide range; but I have not found that the agitation of the water at all affected the formation of these waves.

It is perhaps of importance to state that when these forced waves were being generated, I have suddenly stopped or withdrawn the disturbing point, that the first wave immediately sprang back from the others, showing that it had been in a state of compression—that the ridges became parallel, and moving on at the rate of $8\frac{1}{2}$ inches per second, disappeared in about 12 seconds.

The manner in which the divergence of the ridge passes through the point of disturbance is shown in the annexed diagram. A B is the path of disturbance, the disturbing point being in B; a rod B A is 25 inches long; B C is the diverging wave ridge; a graduated rod A C projects from A B at the point A 25 inches behind B, on which are observed the distance of the wave from A along A C, registered in Col. *b*, Table XXIII.



If a body move with a given velocity along a known line A B, the side A C being measured at right angles to the line of direction A B, and cutting, in C, the line B C which represents the ridge of the wave proceeding from the moving body B; it is required to find the velocity of the wave in the line A D perpendicular to its ridge.

As the triangle A B C is right-angled, $\sin B = \dfrac{\sin A \times b}{\sqrt{b^2 + c^2}}$;

and since the triangle A B D is right-angled, $x = \dfrac{\sin B \times c}{\sin D}$;

hence, the time being the same as that in which A B is described, the velocity is at once obtained.

Table I. contains some observations which were made with a view to the investigation of the ratio subsisting between these velocities. The sides and angles are indicated by the same letters which are used in the diagram.

c, its velocity, and *b* being given; *x*, its velocity, and B were calculated by means of the preceding formulæ.

Table XXIII.

Comparison of Experiments on the Divergence due to given Velocities of Genesis.

Column *c* is the constant measure in inches taken along the path of genesis A B in the figure; the adjacent column is the velocity of genesis along A B in inches per second.

Column b is the length A C, measured by observation in a direction at right angles to A B.

Column x is the length of x deduced from the measure b, and the adjacent column shows the corresponding deduced velocity of the wave at right angles to its ridge.

Column B shows the angles of divergence given by these observations.

Column b' and B′ are numbers corresponding to b and B obtained from the supposition that the velocity of the wave in a direction at right angles to its own ridge is constant and precisely equal to the velocity of the free wave, viz., $8\frac{1}{2}$ inches per second. The deviations of b' and B′ from b and B were chiefly due to disturbance of the fluid produced by the apparatus employed in genesis.

c.	Velocity in inches per sec.	b.	x.	Velocity in inches per sec.	B.	b'.	B′.
					° ′ ″		° ′ ″
25	12	25	17·67	8·49	45 0 0	25·0	45 0 0
25	13	21	16·07	8·37	40 0 1	21·60	40 49 48
25	14·4	17	14·05	8·10	34 13 7	18·27	36 10 26
25	16·0	14	12·16	7·79	29 7 46	15·67	32 5 23
25	18·0	11	10·05	7·28	23 42 55	13·38	28 9 32
25	20·6	9	8·44	6·98	19 44 43	11·31	24 21 24
25	24·0	8	7·61	7·31	17 43 22	9·46	20 44 27
25		7	6·74		15 38 35		
25		6	5·83		13 29 44		

Various considerations induced the acceptance of a constant velocity along x of 8·5 inches per second. The deviations from it in the increasing velocities are due principally to the disturbance of the fluid by the peculiar method of genesis in that instance employed as most convenient. On this assumption the values of b were calculated by the following formula and placed in the column b', and the values of the angle B found in this manner are written under B′.

In the triangle A B D, $\sin B = \dfrac{\sin D \times x}{c}$;

whence in the triangle A C D, $b = \dfrac{\sin D \times x}{\sin C}$.

From what has been said, it follows that there can be no difficulty in calculating the velocity of a body or current from the divergence of the capillary wave.

Let b represent the amount of divergence per foot, the time in which a foot will be described, and consequently the velocity per second, can be obtained by the formulæ which were first given; thus, finding the length of x, and its velocity being known, the absolute time occupied can at once be found, which time is that in which the moving body traverses one foot. In Table II., columns A, B, contain the divergence of the wave expressed in inches per foot, and the corresponding velocity in inches per second.

TABLE XXIV.

For determining the Velocity of Currents or Moving Bodies by Observations of Divergence.

Column A gives the divergence from the path of disturbance measured at right angles to the path, in inches per foot of distance from the disturbing point.

Column B gives the corresponding velocity in inches per second, measured along the direction of the stream or the path of the disturbing point.

Column C contains the angle, which may be observed, at which the wave passes off from the disturbing point and gives in degrees its divergence from the direction of the stream or the path of the disturbing point.

Column D gives the velocity in inches per second corresponding to the angles in C.

A.	B.	C.	D.
12	12·0	60°	9·81
11	12·62	55	10·37
10	13·49	50	11·09
9	14·16	45	12·02
8	15·38	40	13·22
7	17·0	35	14·82
6	19·08	30	17·0
5	22·10	25	20·12
4	27·0	20	24·85
3	35·13	15	32·84
2	51·77	10	48·94
1	102·33	5	97·51
		1	487·10

When the angle of divergence is given, the process is facilitated, as one of the equations used in the previous case has for its sole object the finding of that angle; in Table II., columns C, D, contain the velocities in inches per second corresponding to the given angle of divergence.

Waves of a similar description with those I have here examined, appear first to have been noticed by M. Poncelet, in the course of the valuable experiments made by him and M. Lesbros, which are published in their "Mémoire sur la dépense des orifices rectangulaires verticaux à grandes dimensions présenté, a l'Académie Royale des Sciences," 16th November, 1829. In a notice in the "Annales de Chimie et de Physique," vol. xlvi. 1831, "Sur quelques phénomènes produits à la surface libre des fluides, en repos ou en mouvement, par la présence des corps solides qui y sont plus ou moins plongés, et spécialement sur les ondulations et les rides permanentes qui en résultant," M. Poncelet gives the following description of the phenomena:—

"Lorsqu'on approche légèrement l'extrémité d'une tige fine, formée par une substance solide quelconque, de la surface supérieure d' un courant d'eau bien réglé ou constant, il se forme aussitôt à cette surface une quantité de rides proéminentes, enveloppant de toutes parts le point de contact de la tige et du fluide, et présentant l'aspect d'une série de courbes paraboliques qui s'envelopperaient les uns les autres, et auraient pour axe de symmétrie, ou pour grand axe commun, un droit passant par le point dont il s'agit, et dirigée dans le sens même du courant en ce point. L'extrémité inférieure de la tige occupe le sommet de la première parabole intérieure, qui sert comme de limite commun à toutes les autres; le nombre des rides paraît d'ailleurs être infini, et elles sont disposées entre elles à des intervalles distincts qui croissent avec leur distance au point du contact. . . . . les rides sont parfaitement immobiles et invariables de forme tant que l'état de repos de la tige et de mouvement du courant n'est pas changé; de plus, au lieu de persister plus ou moins après que cette tige a été enlevée, le phénomène disparait brusquement, et a l'instant où le liquide abandonne l'extrémité inférieure de la tige, à laquelle il n'est plus retenue vèrs la fin qu'en vertu de l'adhérence. . . . . le phénomène s'opere essentiellement à la surface supérieure du fluide.

". . . . . . quand le courant se trouve limité per des parois plus ou moins voisines de la tige, et parallèles à la direction générale des filets fluides, le phénomène des rides se reproduit de la même manière et avec des circonstances sensiblement identiques à celles qui auraient lieu si ces parois n'existaient pas, ou si la masse du fluide était indéfinie ; c'est-à-dire que la disposition, la forme et les dimensions des rides sont sensiblement les mêmes, à cela près qu'elles se trouvent brusquement coupées ou interrompues par les parois solides qui limitent le courant comme on le voit représenté, sans éprouver d'ailleurs aucune sorte d'inflexion, de déviation ou de réflexion ; l'action de la paroi n'ayant d'autre effet ici que de soulever, à l'ordinaire la surface générale du niveau du fluide . . . . ! ! . . . . le phénomène des rides se manifeste également à l'entour des corps de dimensions plus ou moins grandes, si ce n'est que ces rides s'étendent plus au loin, sont plus larges, plus saillantes, et forment par conséquent des courbes moins déliées et moins distinctes . . . . . . soit que l'on considère les ondulations dans un même profil, soit que l'on considère les ondulations qui se correspond dans des profils différens ou qui appartiennent aux mêmes rides l'amplitude de ces ondulations, c'est-à-dire leur hauteur verticale sera autant moindre, et l'intervalle qui les sépare d'autant plus grand, que les points auxquels elles appartiennent se trouveront plus éloignés . . . . . ces differens systèmes se superposent exactement aux points de leur rencontres mutuelles sans que leur forme soit aucunement altérée . . . . . l'examen attentif de ces changemens de forme et de position des rides produites à la surface d'un courant quelconque par la presence d'un point fine, serait donc très-propre à faire juger, au simple coup d'œil, de l'état même du mouvement en chacun des points de cette surface, et pour chacun des instants successifs où l'on viendrait l'observer . . . . mais cela suppose qu'on a fait à l'avance ; une étude beaucoup trop compliquée et trop délicate pour que nous ayons pu jusqu' ici nous en occuper . . . . . on trouve, 1°, que les rides sont imperceptibles quand sa vîtesse est moyennement au dessous de 25 c. per seconde ; 2°, qu'elles sont d'autant plus déliées d'autant plus distinctes que la vîtesse est plus grande ; 3°, que le nombre des rides se multiplie aussi à mesure que la vîtesse du courant augmente, surtout aux environs du point du contact de la tige ; 4°, que les longues branches des rides se réservent de plus en plus . . . . quand la vîtesse surpasse 5 ou 6 mètres les differens rides paraissent se réduire à une seule . . . . ce phénomène est telle (in standing water) qu'on croirait volontiers que le déplacement de la tige n'a d'autre effet que de pousser les rides en avant d'elle et d'un mouvement commun sur la surface immobile."

These are mere points of difference between these observations and my own, which I am disposed to attribute to the peculiarities of condition in which the observations of M. Poncelet were made. His observations appear chiefly to have been made in currents, where it was of course impossible to secure uniformity of motion over the whole surface.

Section IV.—WAVES OF THE FOURTH ORDER.

The Corpuscular Wave.

The Sound Wave of Water.

This order of wave I have denominated the corpuscular wave, because the motions by which it is propagated are so minute as to escape altogether direct observation, and it is only by mathematical *à priori* investigation and indirect deductions from phenomena, that we come to recognise its existence as a true physical wave. The motions by which it is propagated are so minute, that it is only by supposing a change in the form of the molecules of the liquid, or of their density, if conceived to be in contact, or an instantaneous and infinitesimal change in the minute distances of the molecules from each other, that the existence of such a wave can be conceived to be possible.

I have not examined this wave by any experiments of my own, and indeed I find that labour to be perfectly unnecessary, for there has been kindly transmitted to me by M. Colladon, a communication of his to the Academy of Sciences, which has been printed in the fifth volume of the "Mémoires des Savans Étrangères," in which there is given in great detail, an account of a complete and most satisfactory determination of the elements of this question.

Newton's approximate determination of the velocity of sound in the atmosphere was followed by that of Dr. Young and M. Laplace, who effected a similar approximation for water and other liquids, and finally the complete solution was satisfactorily given by M. Poisson, the velocity being determined both for solids and liquids by the formula

$$c = \sqrt{\frac{\mathrm{P}k}{\mathrm{D}\iota}},$$

where D is the density of the substance, k the length of a given column, and ι the small diminution of length caused by a given pressure P.

For the determination of the velocity of the sound wave in water, MM. Colladon and Sturm undertook a series of experiments on the compression of liquids, conducted with very ingenious apparatus, and observed and discussed with much accuracy; by this means they obtained values for the quantities P, k and ϵ, from which the velocity of sound should be theoretically determined.

They obtained for the water of the lake of Geneva the following quantities:—

Assuming $D = 1$, $k = 1,000,000$,
they found $\epsilon = 48{\cdot}66$,
and $P = (0^{m}{\cdot}76).g.m = (0^{m}{\cdot}76).(9{\cdot}8088).(13,544)$,
whence $c = 1437{\cdot}8$ mètres,

being the theoretical velocity per second of the sound wave in water.

A very elegant apparatus was next employed for the direct determination by experiment of the truth of this result. Two stations were taken on the lake of Geneva, the mean depth of water lying between them being about seventy fathoms, and the distance between the stations was carefully determined to be 13,487 mètres, or 14,833 yards, about eight miles and a half, lying between the towns Rolle and Thonon. At one end of this station a large bell was suspended at a depth of five or six fathoms below the surface of the water, and struck by mechanism so contrived, as at the instant of striking to explode a small quantity of gunpowder, and so indicate (during a dark night) to the observer, eight miles off, the instant at which the bell was struck. This sound was distinctly heard by a sort of ear-trumpet lowered in the water at the other end, and so the observations made.

The mean time occupied in propagating the sound from one station to the other as thus determined, was nine seconds and a half, or more precisely 9·4 seconds, giving for the velocity of sound by direct experiment

$$c = \frac{13487}{9{\cdot}4} = 1435 \text{ mètres,}$$

the actual velocity of the sound wave thus being found to differ from the theoretical by not three mètres per second.

The velocity of transmission of the wave of the fourth order in water is therefore in English measure about 1580 yards per second, being about one-half more rapid than the velocity of sound through the atmosphere.

DESCRIPTION OF THE PLATES.

PLATE I.

Genesis and Mechanism of the Wave of Translation.—Order I.

Fig. 1. *Genesis by impulsion.*—A X is the bottom of a long rectangular channel filled with water to a uniform depth; P a thin plate inserted vertically in the fluid and fitting the internal surface of the channel. It is moved forward from A towards X through the successive positions P_1, P_2, P_3, P_4, P_5, and heaping up the water before it generates a wave of the first order W_4, W_5, which is transmitted along the channel as at W_5, W_6 to W_8, &c., being transmitted with uniform velocity as a great solitary wave, and leaving the water behind it in repose at the original level.

Fig. 2. *Genesis by a column of fluid.*—In the same channel the movable plate P_7 is fixed so as with the end and sides of the channel to form reservoir A G P_4, containing a column of water G W, raised above the surface of repose of the water in the channel. P_7 is suddenly raised as at P_8 and P_9; the column descends, presses forward the column anterior to P_7 and raises the surface, generating a wave of translation, which is transmitted along the channel as before. After genesis the volume g_1 reposes on the level g_2, the water in the channel having been translated forwards from P to *kk*; every particle of water in the channel has during the transmission of the wave been translated towards X through a horizontal distance equal to P *k*.

Fig. 3. *Genesis by protrusion of a solid.*—L_1 is a solid suspended at the end of the channel, its inferior surface slightly immersed in the fluid. It is suddenly detached, descends, displaces the adjacent fluid, and generates a wave of translation as in the foregoing methods.

Fig. 4 exhibits the phenomena of genesis, transmission and regenesis, or reflexion of the wave of translation.

Fig. 5 exhibits in four diagrams the motions of individual wave particles during wave transmission. The *first* diagram represents by arrows the simultaneous motions of the particles in different portions of the same wave at successive points in its length. At the front of the wave the particles *a*, *c*, *e*, *g*, taken at equal depths below the surface, are at rest. The wave-length is divided into ten equal parts: at the first the motion is chiefly upwards, and very slightly forwards; at the second, less upwards and more forwards; at the third, still less upwards and still more forwards, and so on; the inclination of the path diminishing to the middle of the wave, where the velocity is greatest and the direction quite horizontal. Behind this part of the wave the particles are to be seen descending more and more with a motion gradually retarded, and at the hinder extremity of the wave they are in repose, as at the front. These motions of the particles of water are rendered visible by minute particles of any kind mixed with the water and nearly of the same specific gravity. Such are the simultaneous motions of the successive particles at different stations along the same wave, as observed in a channel by glass windows placed in the sides and carefully graduated in small squares for the purpose of observation, the side of the channel opposite to the window being similarly graduated. The *second* diagram represents the paths of four particles described during the whole period of transmission of a wave. The wave is transmitted from A towards X. The anterior extremity of the wave finds one particle at *a* and carries it forward through an ellipse to *b*, where it is left by the end of the wave: the same wave translates the particle *c* vertically below *a* through its elliptical path and leaves it at *d* vertically below *b*, and in like manner *e* and *g* are transferred to *f* and *h*. All these paths are semi-ellipses (as nearly as it is possible to observe them), and are of the same major axis; but the semi-minor axis is at the surface equal to the height of the wave-crest, and diminishes with the distance from the bottom of the channel, where it is nil. The *third* diagram exhibits the phenomena of vertical sections during wave transmission: small globules of greater specific gravity than water are suspended at different depths by means of long slender stalks of less specific gravity. These globules are arranged while the water is in repose, in vertical planes at equal distances along the fluid. These vertical planes are, by transmitting the wave, made to approach each other, but still retaining their verticality without sensible disturbance. At the middle of the wave-length they are brought closest, and at the hinder extremity they recede and settle down at their original depth. The *fourth* diagram shows the change of the position of points in the same horizontal planes during wave transmission, particles vertically equidistant in repose remaining equidistant during wave transmission.

Fig. 6. *Genesis of Compound waves.*—The first diagram represents the genesis by a large low column of fluid of a compound or double wave of the first order, which immediately breaks down by spontaneous analysis into two, the greater moving faster and altogether leaving the smaller. The second diagram represents the genesis by a high small column of fluid of a positive and negative wave, which soon separate, the positive wave travelling more rapidly, leaving altogether the residuary negative wave. The negative wave is further noticed in another Plate. W_1 is the positive and w 1 the residuary positive or negative wave as generated. W_2 and w_2 represent them separated by propagation.

PLATE II.

Discussion of Observations on the Velocity of Waves.—Order I.

Fig. 1. Comparison of the observations marked by stars with the formula B, indicated by the parabola A B, of which A X is the axis, parallel to which are measured abscissæ I., II., III., &c., representing the depth of the fluid in inches, the corresponding velocities being represented by ordinates A 1, A 2, A 3, A Y, &c., at right angles to A X. The manner in which the curve passes through among the stars, shows the close approximation of the results of individual experiments to the formula B adopted to represent them. These are taken from the Table V.

Fig. 2 exhibits a similar comparison for waves of a larger size than the former. See Table IV.

Figs. 3 and 4 show a comparison with the observations, marked by stars as before, with formulæ proposed by Mr. Airy, shown as dots connected by dotted lines, and with the formulæ employed by the author, shown by a continuous black line A B. The eye at once decides whether the black line or the dots and dotted line most nearly coincides with the stars. See Tables VI. and VII.

Fig. 5 exhibits a similar comparison of the velocity of negative waves, as observed in a rectangular channel along A B, and in a triangular channel as shown along A B′. The stars show that the velocity falls below that which the formulæ would assign as due to the depth, especially in the triangular channel. See Tables XI. and XII.

Fig. 6 exhibits the general results of experiments on velocity; the horizontal spaces indicate depths of five inches to each, and the velocities in per second are represented by the vertical spaces which represent each the velocity of one foot per second in transmission of the wave. A B is the line of the formula, for a rectangular channel, see Table III.; and A B′ for a triangular channel, see Table XV.

Fig. 7. A X is the surface of water four inches deep; B X represents the successive heights of a wave as referred to in Table II.

Plate III.

Rediscussion of the Experiments on Velocity.—By the Method of Curves.

Fig. 1. B C, D E, F G, &c., are lines drawn by the eye through the observations of heights of waves shown by the stars; similar lines were drawn through the corresponding observations of velocity. These waves were taken as representing the experiments cleared of errors of observation; they were then collected and laid down in fig. 2.

Fig. 2. A B is the line given by the formulæ employed by the author to represent the velocity of the wave of the first order; the stars are the observations freed in some measure from errors of observation as described above.

Plate III. (continued).

Effects of Form of Channel on the Wave.—Order I.

Note.—In a rectangular channel on a level plane the crest of the wave is a horizontal line, parallel to the bottom.

Fig. 3. The section across a channel; *a w* the surface of the water in repose; $a\ d = 4$ inches; $w\ e = 1$ inch; A *a* the height of the wave-crest $= 1\frac{1}{2}$ inch; B *w* the height on the shallow side $= 2\frac{1}{2}$ inches.

Fig. 4. A B *d* the cross section of a triangular channel, A B the crest of the wave, *a w* the level of the water in repose; the angle A *d* B = 90°.

Fig. 5. B *c d* a slope of 1 in 3, being the cross section of a channel *c d f;* A B the crest of the wave breaking on the sides, where the height of the wave becomes equal to the depth of the water.

Fig. 6. Cross section of another form of channel.

Fig. 7. The sea-beach near Kingstown and Dublin. Common sea-waves, W_1, W_2, W_3, W_4, W_5, W_6, break on the ridge *d*, where their height is equal to the depth of the still water. They generate small waves of the first order, w_1, w_2, w_3, w_4, w_5, &c., which are propagated through the still, shallow water to great distances, and the intervals between them are left level and in repose.

Plate IV.

Waves of the First Order.—Drawn by themselves.

These eight waves are of the natural size, being mere transcripts of the outline of a wave left on a dry surface. The four lower outlines in the Plate were obtained by inserting a dry surface, moved horizontally with a uniform velocity equal to that of the wave, and instantly removing it. The moist outline left by the wave was copied on tracing paper, and transferred without change to the copper-plate. Another method produced the four upper outlines, which were obtained by passing under the wave to be observed another wave transmitted in the opposite direction. These outlines are not therefore to be regarded as copies of a wave, but as transcripts of the outline left by the passage of one wave over another; the crests of both describe horizontal straight lines on the side of the channel, but as every point of one may be regarded as passing over the crest of the other, there is a moist outline left on the side of the channel at the crossing, which outline is simply transferred to the copper, as in the four upper waves. Where a dotted line occurs a blank was left in the outline, which is filled up by the eye. The depth of the water was 2 inches, and the parallel lines in the figure are at 1 inch apart and serve as a scale.

Plate V.

These waves are taken in the same manner, but have been reduced from the original outlines to a smaller scale—smaller than the original in the ratio of 2 to 3. The horizontal lines are $\frac{2}{3}$rds of an inch apart, which represents an inch on the full size. The four lowest are taken from waves in water 2 inches deep on a sloping beach, parallel to *g* X, *k* X, *l* X, and *m* X, with an inclination of 1 in 12. The four next are imperfect or compound waves, taken from the outline left by passing another in the opposite direction. The two highest are taken in the same way, one of them in the act of breaking.

Plate VI.

The Wave of the First Order.

Fig. 1 represents the genesis of a compound wave by impulsion of the plate with a variable force and velocity, which variations have produced corresponding variations on the wave form. After propagation the wave breaks down by spontaneous analysis; the higher part moves forward, as shown by the dotted line, and ultimately leaves the rest behind, so that after the lapse of a considerable period the compound wave is resolved into single separate waves, each moving with the velocity due to the depth.

Fig. 2 represents the phenomena resulting from genesis by a long, low column of water. Instead of genesis of a compound wave, as in the former case of impulsion, the added mass sends off a series of single waves, the first being the greatest: these however do not remain together, but speedily separate, as shown in the dotted lines, and become the further apart the longer they travel.

Figs. 3, 4, 5 and 6 give geometrical approximations to the representation of the wave form and phenomena. In fig. 3, d D d is the length of a small wave divided into ten equal parts; $c\ d$ is equal to the height of the wave, on which a circle is described, and of which the circumference is also divided into ten equal parts. Through these equal divisions of the circle are drawn horizontal lines, which are intersected by vertical lines from each of the divisions of the straight line $d\ d$, as shown in the figure. A continuous line, passing through these points of intersection, has for its vertical ordinates the versed sines of the arcs of the circle, while its abscissæ are proportional to the arcs themselves. Such a line is the curve of versed sines, and gives a first approximation to the form of the wave of the first order.

Fig. 4 gives a second approximation to the form and the representation of the phenomena of the wave of the first order. A D d is taken equal to the length of the wave in the first approximation = 6·28 times the depth of the fluid in repose; on $d\ c$ = the height of the wave, a circle is described and divided into equal arcs as formerly, and thus a dotted line, A C d, formed as before, being the first approximation to the wave form. These equal arcs being taken to represent equal times, the versed sines also represent the rise and fall of the surface of the wave during equal successive intervals of time. But hitherto we have neglected the motion of translation, the horizontal transference of each vertical column of fluid in the direction of wave transmission simultaneous with the vertical motion. Take the length A to A′, such that A A′ × A B shall $= \frac{V}{b}$ = the volume of water generating the wave divided by the breadth of the fluid. This length, A B, in a small wave will be about three times the height of the wave. Take A A′ as the major axis of an ellipse, of which the minor axis is C D or $c\ d$, the height of the wave. Let the horizontal lines through the equal arcs of the small circle $c\ d$ be extended to pass through the ellipse A A′, and from the points of division let fall perpendiculars on A A′ on the points 1, 2, 3, 4, 5, 6, 7, 8, 9, then the lines on the axis A A′, viz. A 1, A 2, A 3, A 4, A 5, A 6, A 7, A 8, A 9, A A′ represent the amount of horizontal transference effected during the same time, in which a given particle on the surface is rising and falling through the versed sines of the equal arcs, viz. *d* 1, *d* 2, *d* 3, *d* 4, *d* 5, *d* 6, *d* 7, *d* 8, *d* 9, *d d*. Let us now effect this horizontal transference on each point of the surface on the first wave A C *d*, by advancing the point 1 horizontally through a distance equal to A 1; 2 through a distance A 2; 3 through a distance *a* 3, and so on, and we shall get a curve A′ C′ *d*, which closely represents the form of the wave, and also its phenomena of horizontal translation = throughout the whole depth to A 1, A 2, A 3, A 4, A 5, &c.

Fig. 5 is obtained in the same way as fig. 4, only for a larger wave; where the height is nearly equal to the depth of the fluid, the ellipse is nearly a semicircle. The same ellipse represents also the absolute path of a particle on the surface during wave transmission. Ellipses of the same major axes, but having their minor axes diminishing with their distance from the bottom of the channel, will represent the simultaneous motions of particles below the surface.

Fig. 6 shows a single particle path, and three successive positions of the wave outline in regard to it. The figures 1, 2, 3, 4, 5, &c., give the simultaneous positions of the particle referred to the wave surface, and the same particle referred to the path of the particle. When at 1, 2, 3, 4, 5, &c., in the orbit, the particle is also at 1, 2, 3, 4, 5, &c., in the wave surface. Thus the points which succeed each other towards the right on the path, succeed towards the left on the wave form.

Figs. 7 and 8 represent the genesis of the negative wave of the first order. A solid Q 2 reposes suspended in the fluid, and is suddenly raised out of it. A negative wave is generated and propagated along the channel, as W 1 in figs. 8, 9, and 10. This negative wave of the first order, if it encounter a positive wave of the first order, of equal volume, does not pass over it, but they neutralise each other and are annihilated. If unequal, their difference, positive or negative, alone remains, and is propagated as a wave of the first order.

Figs. 9 and 10 record observations, showing that although the negative wave is in its own order solitary, yet that its existence is the cause of genesis of a group of gregarious waves, or waves of oscillation of the second order; W.1 is a negative wave of the first order: W 1, W 2, W 3, &c., are all waves of the second order. The curved arrows in fig. 9 show the semi-elliptical path of the particles during the transmission of the negative wave. After which, during the transmission of the waves of the second order, the particles describe circles, continually diminishing in diameter as the waves gradually subside.

PLATE VII.

Waves of the First Order.—Reflexion, Non-reflexion and Lateral Accumulation.

In this Plate a wave of the first order, W_1 R, is represented as incident upon a vertical plane surface immovable at R; *i.e.* the ridge of the wave forms a given angle R_1 W A. After impact at R the wave is reflected, so that the angle of reflexion is equal to the angle of incidence; and when the angle of direction of transmission is great (*i.e.*, when the angle of the ridge with the surface is small, not greater than 30°), the reflexion is complete in angle and in quantity. When the angle of direction of transmission diminishes (*i.e.* when the ridge of the wave makes an angle greater than 30°), the angle of reflexion is still equal to the angle of incidence, but the reflected wave is less in quantity than the incident wave. The magnitude of the reflected wave diminishes as the angle of incidence diminishes, until at length, when the angle of the ridge of the wave is within 15° or 20° of being perpendicular to the plane, reflexion ceases, the size of the wave near the point of incidence and its velocity rapidly increases, and it moves forward rapidly with a high crest at right angles to the resisting surface. Thus at different angles we have the phenomena of total reflexion, partial reflexion, and non-reflexion and lateral accumulation; phenomena analogous in name, but dissimilar in condition from the reflexion of heights, &c.

Plate VIII.

Lateral Diffusion of the Wave of Translation round an Axis.

Figs. 1, 2, 3, and 4 represent a large rectangular reservoir of water filled to a uniform depth with water. It is 20 feet square. From a chamber C in one corner a wave of the first order was transmitted in the direction W 1, W 2; and the observations made which appear in the figures.

In fig. 4 the aspect of the phenomenon is represented. The wave is propagated in the direction of original propagation, which we shall call its axis, with a gradual diminution of its height according to the length of its path along the axis. The observations are probably not yet sufficiently numerous to determine accurately the law of diminution. From this axis the wave spreads on every side. At right angles to the axis of propagation the height of the wave is scarcely sensible, and the diminution of magnitude is very rapid as the line of direction diverges from the axis. The wave is also propagated faster in the direction of the primary axis than in any other direction, so that the wave-crest is elliptic and elongated in that direction.

In fig. 3 the heights of a wave are marked by lines. Each line along W *w* and W 2 *w* represents one-tenth of an inch in height of the wave; so that the height of the wave is indicated to the eye by the number of lines. These observations are made on concentric circles.

In figs. 1 and 2 the same kind of observations is represented only along straight lines.

Plate IX.

Waves of the Second Order.—Standing Waves in Running Water.

The forms of the waves in these figures are the same as those in figs. 9, 10 of Plate VI., being all cycloidal; with this difference only, that the waves in Plate VI. were moving along the standing water with a uniform velocity, while those in Plate IX. are standing in the running water. The generating course in this case is a large obstacle or large stone in the running stream. On this the water impinges; it is heaped up behind it; it acquires a circular motion which is alternately coincident with and opposed to the stream; the water having once acquired this circular oscillating motion in a vertical direction retains it, the water is alternately accumulated and accelerated, and thus standing waves are formed, as shown in figs. 1 and 2.

Figs. 3 and 4 exhibit a remarkable case of the coexistence in one stream of two sets of waves moving with velocities differing in about the proportion of two to three. On one side of a stream there projected a ledge of rock M, over which fell a thin sheet of water into a large pool, nearly still, without generating any sensible wave. On the opposite side a deep violent current was running round the obstacle with great rapidity. The middle part of the channel was occupied by a large boulder, over which also a stream flowed, generating standing waves with a smaller velocity. These waves are also remarkable for non-diffusion, as they will preserve their visible identity to a great distance without being dissipated.

Plate X.

Waves of the Second Order.—Their Mechanism.

All the waves of the second order, whether standing waves in running water or travelling waves in standing water, exhibit the forms of the curves B A B C D in fig. 1. These are cycloids, having for their base the rectilineal distance A C, and for their height the corresponding circles. In the case of standing waves in running water these cycloids represent the actual paths of individual particles of water in the running stream, as shown in Plate IX. In the case of travelling waves in standing water, the circles represent the paths described by the individual particles of water, and the cycloids the visible moving surface presented to the eye. The motion of oscillation in the *upper* half of the circle is in running water, *opposite* to the motion of the stream, and in standing water is in the same direction as the visible motion of transmission of the waves. The figure shows the rapid diminution of the motion of oscillation with the depth. I am indebted for this figure to M. Gerstner, whose theory it illustrates, and I have given it because I find it represent my own observations as correctly as any figure of my own could do. I have only found it necessary in reconstructing his figure to clear it of some slight inaccuracies. The shaded parts on the left show the different forms which given portions of water successively assume during wave motion. The circular orbits are divided into equal portions, numbered 1, 2, 3, 4, &c., to show that the particles of water are in those points of the circles at the same instants the corresponding particles are at the points 1, 2, 3, 4, &c., of the cycloidal paths.

PLATE X. (continued).

Waves of the Third Order.—Capillary Waves.

Fig. 2 represents a slender rod inserted in standing water, raising round it by capillary attraction a circular portion of the surface of the fluid. A slow motion gives it the form represented in fig. 3, and more rapid motions, but all of less than a foot per second, give it the forms in figs. 4, 5, and 6; at the velocity of one foot per second the phenomena become those represented in Plate XI.

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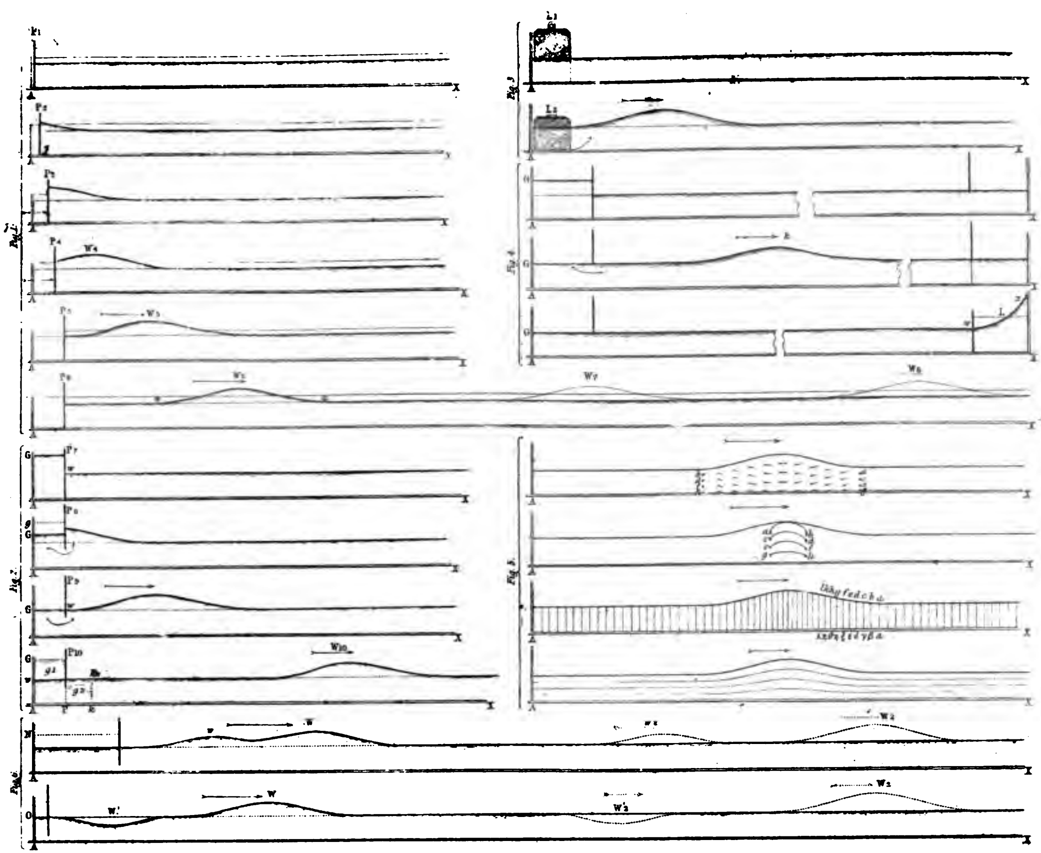
WAVES_Order I. The Great Wave of Translation



PL. II.

WAVES _ Order 1. Discussion of Experiments

Fig. 1.

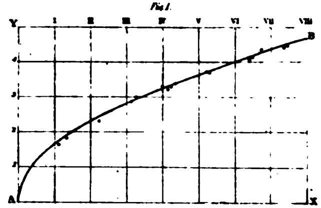


Fig. 2.

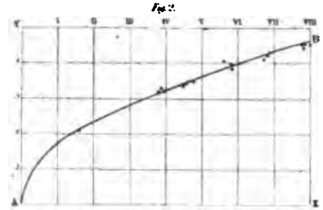


Fig. 3.

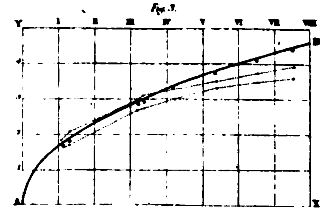


Fig. 6.

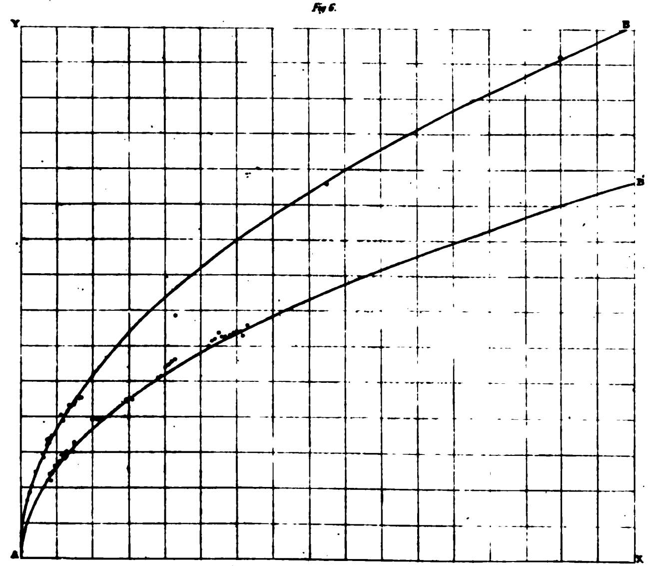


Fig. 4.

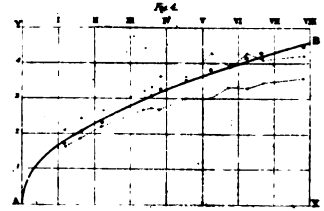


Fig. 7.

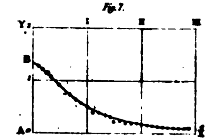
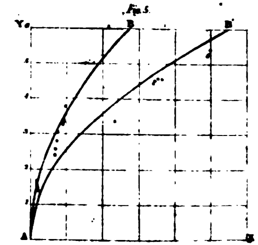


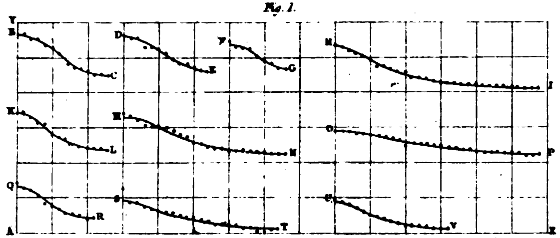
Fig. 5.



WAVES __ Order 1.

Re discussion of the Experiments.

Fig. 1.



Effects of form of Channel.

Fig. 3.

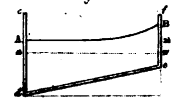


Fig. 4.

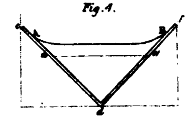


Fig. 2.

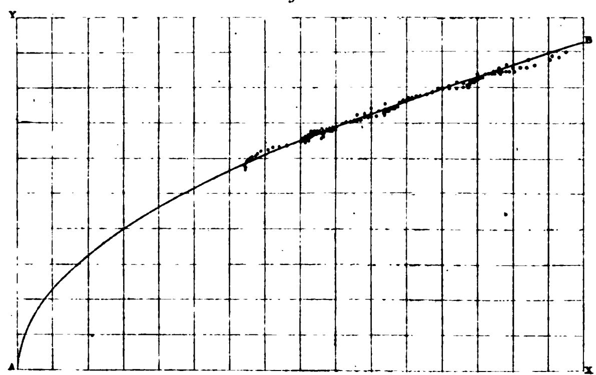


Fig. 5.

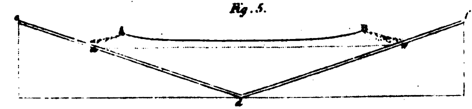
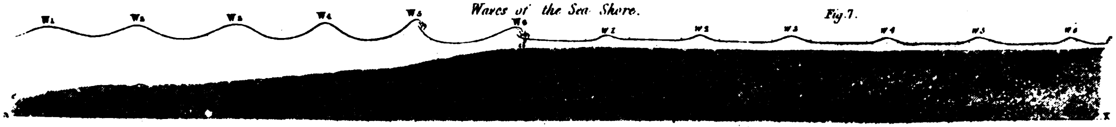


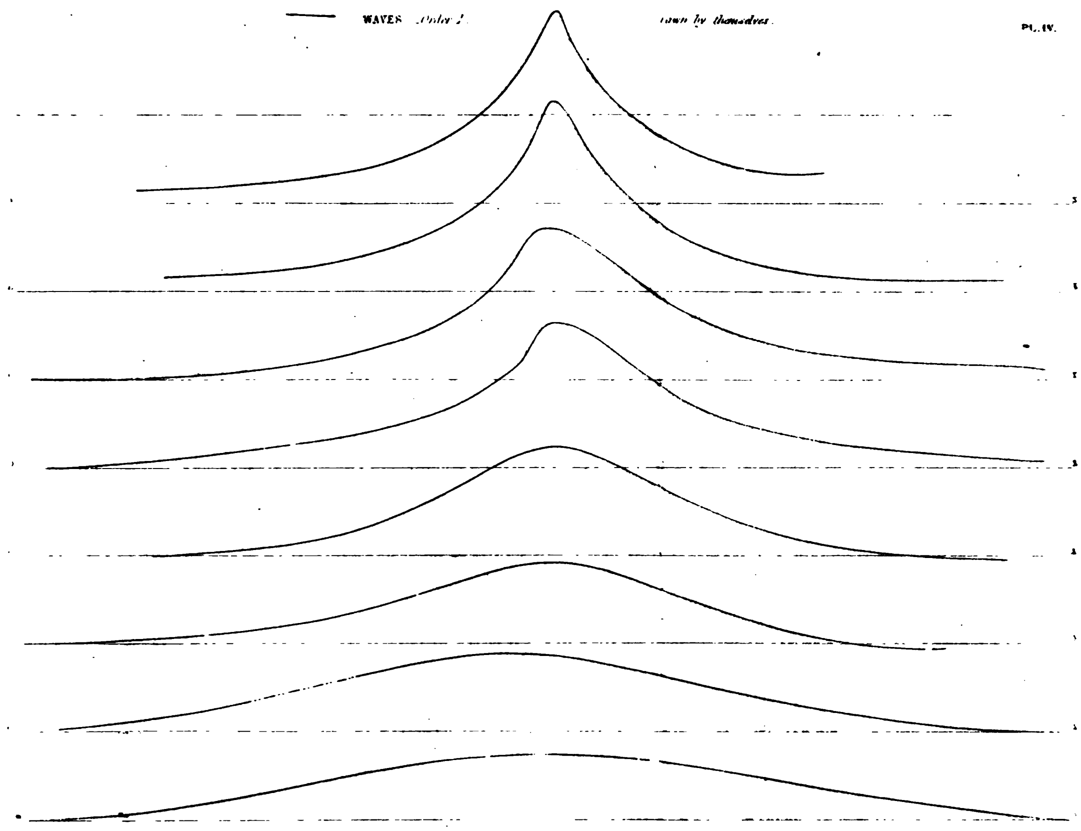
Fig. 6.

Waves of the Sea Shore.

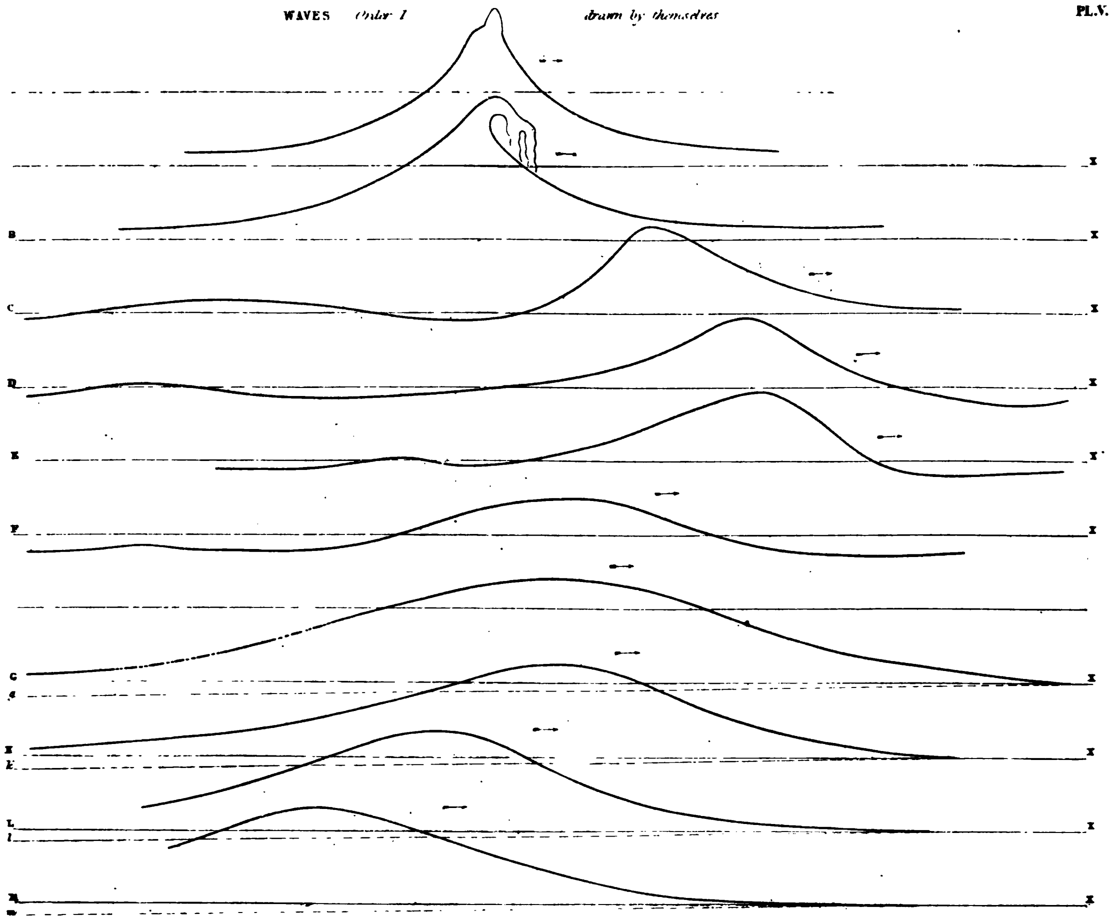
Fig. 7.



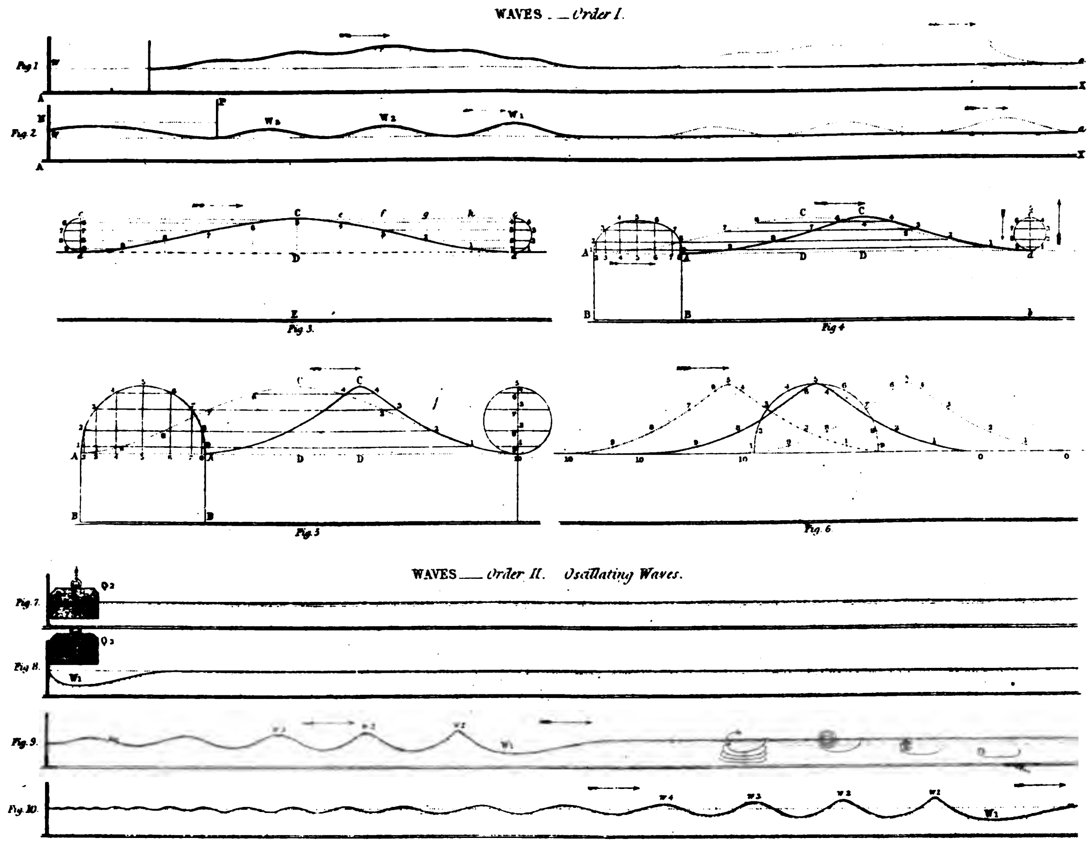
WAVES (Order 2) ... own by themselves. PL. IV.



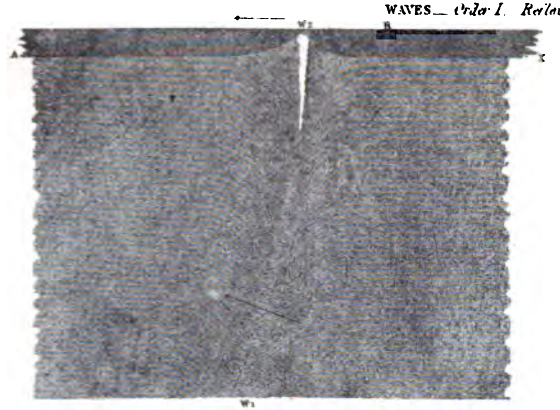
WAVES Order I drawn by themselves PL. V.

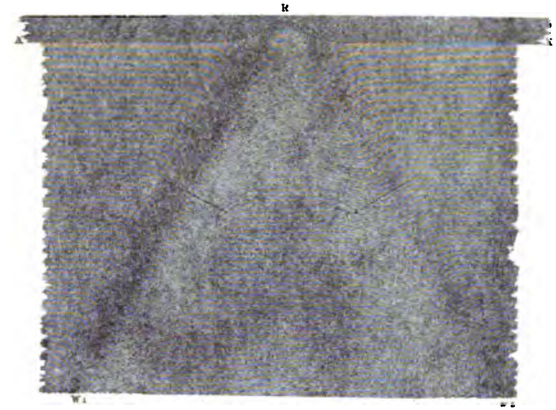


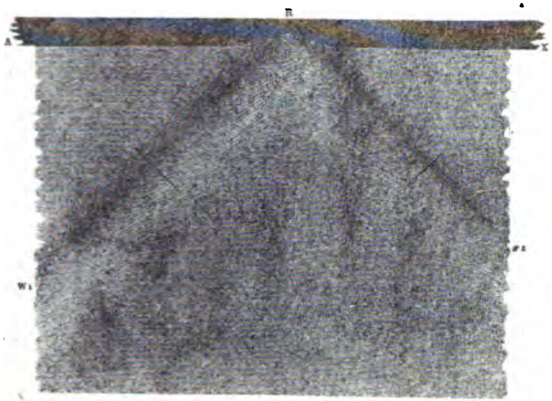
WAVES.—Order I.

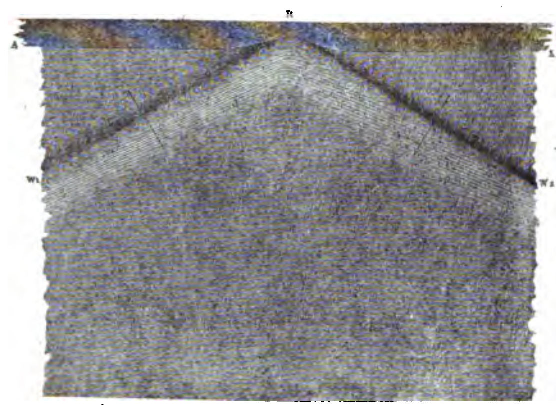


WAVES — Order I. Reflexion & lateral accumulation. PL. VII.









WAVES. Order I.

Fig. 1.

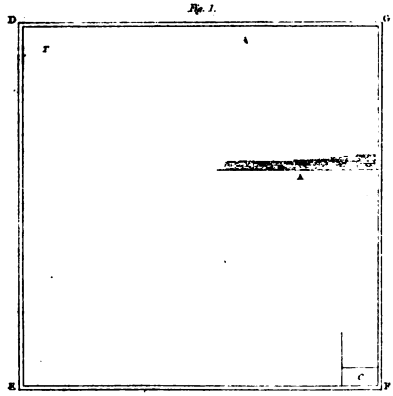


Fig. 3.

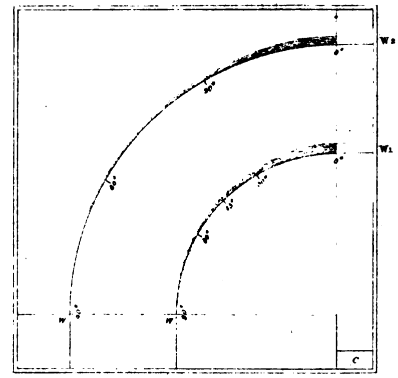


Fig. 2.

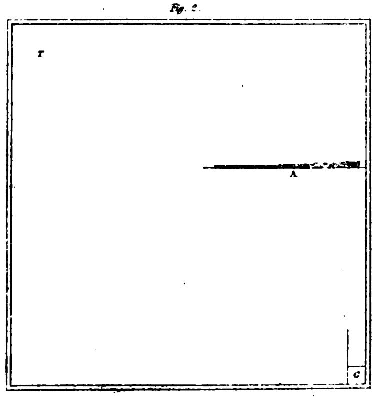
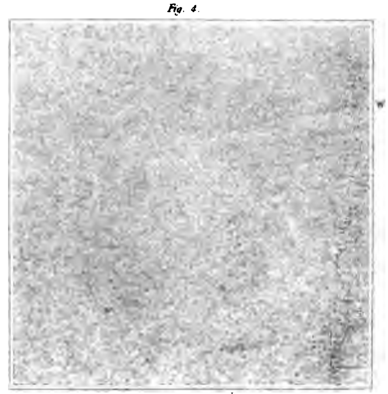


Fig. 4.



WAVES — Order II — Standing Waves.

Fig. 1.

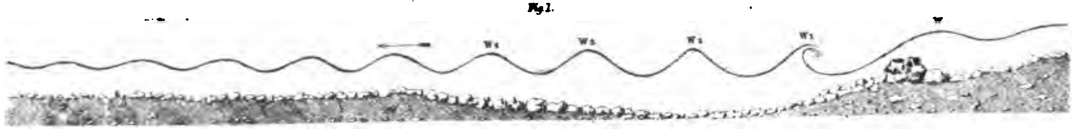


Fig. 2.

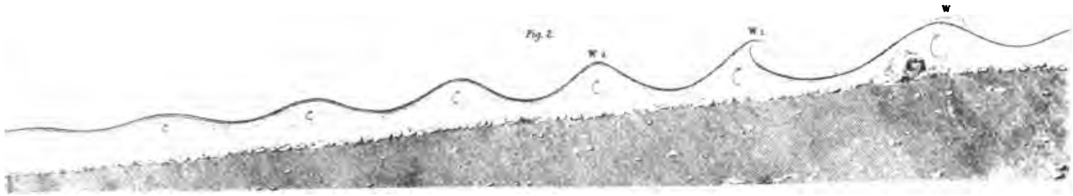


Fig. 3.

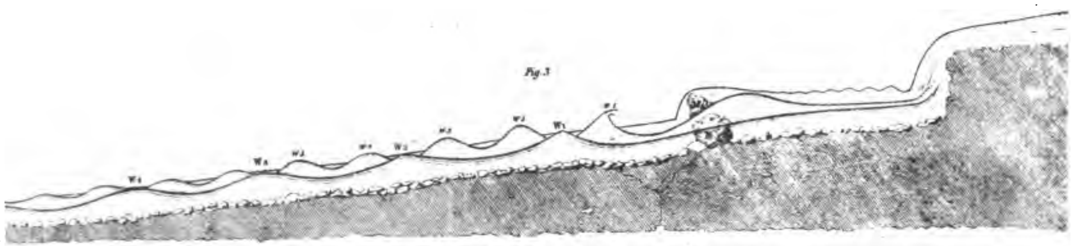
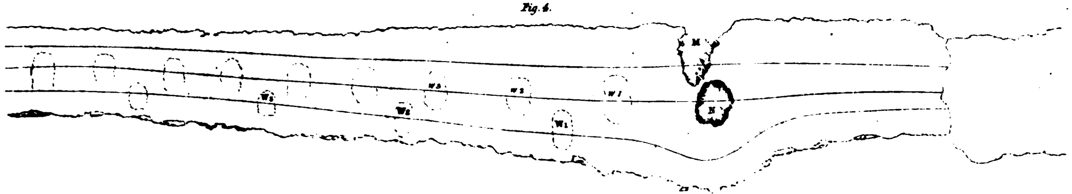


Fig. 4.



PL X.

WAVES — Order II.

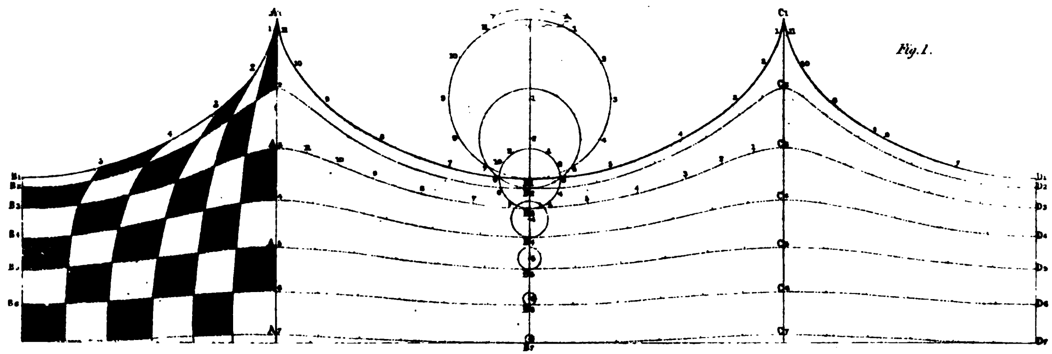


Fig. 1.

WAVES — Order III.

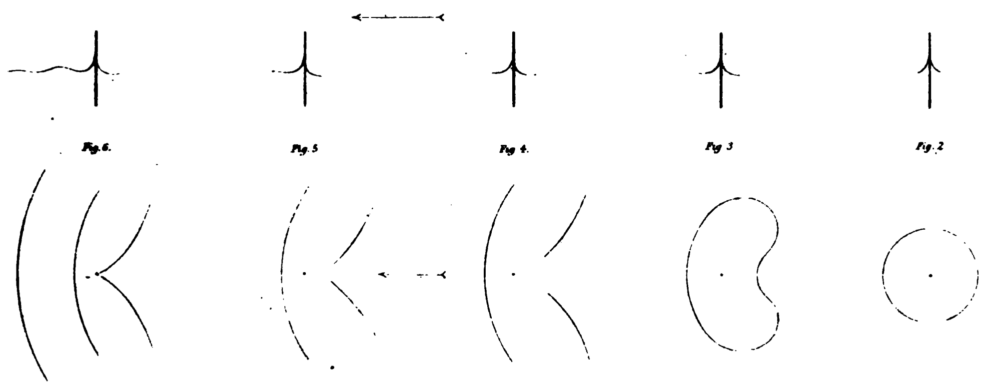


Fig. 6. Fig. 5. Fig. 4. Fig. 3. Fig. 2.

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